

QUALITY OF INFORMATION AND OLIGOPOLISTIC PRICE DISCRIMINATION

QIHONG LIU

*Department of Resource Economics
University of Massachusetts
Amherst, MA 01003-9246
qiliu@resecon.umass.edu*

KONSTANTINOS SERFES

*Department of Economics and International Business
Drexel University
Philadelphia, PA 19104
konstantinos.serfes@drexel.edu*

Recent developments in information technology (IT) have resulted in the collection of a vast amount of customer-specific data. As IT advances, the quality of such information improves. We analyze a unifying spatial price discrimination model that encompasses the two most studied paradigms of two-group and perfect discrimination as special cases. Firms use the available information to classify the consumers into different groups. The number of identifiable consumer segments increases with the information quality. Among our findings (1) when the information quality is low, unilateral commitments not to price discriminate arise in equilibrium; (2) after a unique threshold of information precision such a commitment is a dominated strategy, and the game becomes a prisoners' dilemma; and (3) equilibrium profits exhibit a U-shaped relationship with the information quality.

1. INTRODUCTION

The rapid development of the Internet as a medium of communication and commerce has enhanced the firm's ability to accumulate

A previous version of this paper was titled "Endogenous Acquisition of Information on Consumer Willingness to Pay in a Product Differentiated Duopoly." We would like to thank a coeditor and two referees for very helpful comments and suggestions. We also would like to thank Olivier Armantier, George Deltas, Pradeep Dubey, Debra Dwyer, John Hause, Charlie Kahn, Timothy Mathews, Preston McAfee, Sangin Park, Martin Perry, Greg Shaffer, Richard Steinberg, Andrew Stivers, Yair Tauman, and seminar participants at the Western Economic Association Conference in San Francisco, July 2001, the 12th Annual Conference on Game Theory at Stony Brook, the Summer 2002 Meetings of the Econometric Society at UCLA, and the 2003 International Industrial Organization Conference at Northeastern University, Stony Brook and Rutgers. The usual disclaimer applies.

© 2004 Blackwell Publishing, 350 Main Street, Malden, MA 02148, USA, and 9600 Garsington Road, Oxford OX4 2DQ, UK.

Journal of Economics & Management Strategy, Volume 13, Number 4, Winter 2004, 671-702

a vast amount of customer-specific information. Recent advances in information technology (IT) and software tools, coupled with all this accumulated data, have taken price discrimination to a new level. The practice of dynamic pricing, where consumers pay different prices depending upon their demographics, purchasing history, and income, is one prominent example.¹ Another example is coupon targeting. Consumers are asked regularly, when they visit a retailer's web page, to register by divulging personal information such as their name, email, address of residence (or zip code), age, and family size. This, in turn, can facilitate the distribution of targeted coupons via email with different face values based upon each customer's willingness to pay, which is implied by his frequency of past coupon redemptions combined with his personal characteristics.² Information technology also allows sellers to keep track of consumers' purchasing behavior over time and to merge an increasing number of seemingly unrelated databases, which implies that the available customer data can be updated and refined easily.

We develop a location model of oligopolistic third-degree price discrimination to study: (1) the incentives of firms to acquire customer-specific information of a given level of quality; and (2) the evolution of these incentives, profits, and welfare as the quality improves. The available information—which is modeled as a partition of the characteristic space—allows firms to classify the consumers into different segments by imperfectly estimating their degree of brand loyalty. Firms then can tailor their prices to each consumer segment. Higher information quality is modeled as a refinement of the partition.

The vast majority of the literature on spatial price discrimination is based on one of the following two *extreme* assumptions: Either (1) firms have the ability to identify the location of each consumer *perfectly* (e.g., Anderson and de Palma, 1988; Bhaskar and To, 2004; Lederer and Hurter, 1986; Shaffer and Zhang, 2002; Thisse and Vives, 1988); or (2) they are able to discriminate between only *two* groups of consumers (e.g., Bester

1. All one has to do is type the phrase *dynamic pricing* in google.com and a plethora of relevant links will appear. Also, see the article "On the Web, Price Tags Blur: What You Pay Could Depend on Who You Are," washingtonpost.com, September 27, 2000. In the same article it is stated that: Amazon, the largest and most potent force in e-commerce, was recently revealed to be selling the same DVD movies for different prices to different customers. Bailey (1998) offers another example of behavior-based price discrimination: Books.com—a books retailer—adopted in early 1998 a price discrimination strategy where different buyers were paying different prices for the same item depending on their shopping behavior.

2. See Rossi et al. (1996) and Allenby and Rossi (1999), who model consumer heterogeneity and develop statistical procedures, based on panel data on household purchase behavior, in order to estimate consumers' sensitivity on (among other things) targeted price promotions.

and Petrakis, 1996; Chen, 1997; Corts, 1998; Fudenberg and Tirole, 2000; Shaffer and Zhang, 1995, 2000).³

Nevertheless, one important question remains unanswered: how do the profits and welfare evolve as the sellers' ability to segment the consumers gradually improves? Our main innovation is in extending the literature by introducing a unifying framework that has the aforementioned two extremes as its two limit points and is capable of handling all the cases of price discrimination facilitated by a partition of the characteristic space that lies anywhere between these two polar partitions.⁴ This modeling approach offers a more comprehensive picture of the information acquisition incentives and the transition of the equilibrium variables. More importantly, it provides a closer approximation of reality where the quality of consumer information firms are utilizing to develop their pricing strategies is far from perfect but is improving constantly due to advances in IT.

In our model the information provides a credible commitment technology. If a firm does not acquire information, then it is not feasible to practice price discrimination. We show that for very low levels of information quality, unilateral commitments to a uniform price arise in equilibrium (even with a zero cost of information). After a unique threshold of information quality, however, such a commitment is not an equilibrium. Acquiring information becomes each firm's dominant strategy, resulting in lower profits than the ones obtained under a uniform pricing rule. We should expect, then, that as the quality of information about their customers' preferences improves, firms will abandon, once and for all, policies that aim at limiting the practice of price discrimination.⁵

Interestingly, the profits when both firms price discriminate are nonmonotonic as a function of the information precision. In particular, they exhibit a U-shape pattern. This indicates that better information initially *intensifies the competition* between the sellers but that eventually the *surplus extraction* effect prevails and firms become relatively better off when the information is refined further. Our nonmonotonicity result stands in stark contrast with the one obtained by Chen et al. (2001), who

3. A notable exception is the paper by Kats (1987), where the number of consumer segments varies. In particular, there are no informational constraints, as both firms have perfect knowledge of each consumer's location. The constraint is in the number of different prices the firms can offer. Firms are allowed to charge only a finite number of prices. Customer segmentation then is chosen by the firms in a way to ensure existence of a pure strategy equilibrium. As the number of distinct prices grows (exogenously), the profits monotonically decrease, and they converge to the perfect discrimination equilibrium profits. This modeling framework, however, does not conform readily with the practice of price discrimination based on imperfect information, which is the focus of our paper.

4. See section 2 for an exact definition of the sequence of partitions we consider.

5. Such as everyday low pricing and no-haggle policies.

show that the firms' equilibrium profits exhibit an inverted U-shape as a function of the information accuracy (targetability). We elaborate more on this comparison in section 5.

Policymakers and regulators have raised concerns that the increasing collection of information about consumers' shopping behavior may have detrimental effects on consumer welfare. Consumer groups and organizations are concerned also about the way that personal information collected from consumers or about consumers by third parties is used.⁶ This paper attempts to highlight some of these issues.⁷ We show that consumer welfare has an inverted U-shape relationship with the information precision, implying that moderate information quality is the most beneficial for the consumers. After the peak of consumer welfare, as the information quality improves some consumers start paying (relatively) higher prices.

The rest of the paper is organized as follows. The model and the three-stage game are presented in Section 2. The game is analyzed in Section 3. Section 4 offers a discussion of the main results and places them in the existing literature. We conclude in Section 5. The two main proofs can be found in the Appendix.

2. THE DESCRIPTION OF THE MODEL

Two firms, 1 and 2, located at the two endpoints of a unit interval sell competing brands to a continuum of customers who have unit demands and are uniformly distributed on $[0, 1]$.⁸ We assume that each consumer derives a benefit equal to V if she buys a product from either one of the firms. Let p_1 and p_2 be the prices that firm 1 and 2 charge, respectively. Both firms' marginal costs are normalized to zero. In addition, each consumer incurs a linear unit transportation cost denoted by $t > 0$. Therefore, a consumer who is located at point $x \in [0, 1]$ and buys from

6. See, for instance, the Electronic Privacy Information Center's (www.epic.org, August 10, 2001) complaint against Microsoft concerning Windows XP and Microsoft's ability to collect a huge amount of personal information that is allegedly unfair and leads to deceptive trade practices. See also the investigation launched by the Federal Trade Commission (FTC) against DoubleClick Inc., an Internet advertising company, about whether in collecting and maintaining information concerning Internet users the firm has engaged in unfair or deceptive practices ("DoubleClick Target of FTC Inquiry," USATODAY.com, June 7, 2000).

7. Of course, the whole debate about consumer privacy and the issue of the proprietary rights of consumer information and how this information should be used is far more general than the specific approach we have taken in this paper.

8. The uniform distribution renders the equilibrium solvable in closed-form expressions, which enables us to characterize the solution to the entire game in a very transparent way (see, for example, Thisse and Vives, 1988, section 3, for a similar approach). In note 17, we offer a short discussion on how far the model can be pushed without this assumption.

firm 1 enjoys a surplus of $V - tx - p_1$. Likewise, if she buys from firm 2, her surplus is $V - t(1 - x) - p_2$. Each consumer buys the product that gives her the highest positive surplus. We assume that V is sufficiently high, ensuring that each consumer will buy.

Up to this point, we have assumed that all the firms know about consumers is that they are uniformly distributed on the unit interval. Now assume that (some) information about the location of each consumer becomes available. In practice, firms can obtain such information from a number of different sources, such as (1) directly, through repeated past transactions with the customers; (2) via a telemarketing or direct-mail surveys; (3) from credit card reports; or (4) from a marketing firm [see Shaffer and Zhang (2000, 2002) for a more extensive discussion and more references on this issue].⁹ The information partitions the $[0, 1]$ interval into N subintervals (indexed by m , $m = 1, \dots, N$) of equal length, and the firm who acquires this information obtains a more accurate estimate of how much each consumer values its product. In this case, a firm can charge different prices (p_{im} , $i = 1, 2$ and $m = 1, \dots, N$) to different groups of consumers (by, say, distributing targeted coupons via email), though the price is the same within each group. Arbitrage between consumers is not feasible.

We further assume that $N = 2^k$, $k = 0, 1, 2, \dots$. Hence, k will parameterize the information quality, with higher k 's being associated with higher information precision (information refinement).¹⁰ We assume that an information of quality k is available to both firms at an exogenously given and sufficiently low price and that the current state of technology dictates k , which the firms take as exogenously given.¹¹ Hence, our model is static, and the effect of information improvements on the equilibrium of the game is in the form of a comparative statics analysis.

9. One example is the *Abacus Catalog Alliance*, a database that contains transactional data with detailed information on consumer and business-to-business purchasing and spending behavior. It is a blind alliance of 1,800 merchants offering shared data representing over 90 million households and is the largest proprietary database of consumer transactions used for target marketing purposes (see <http://www.doubleclick.com/us/>).

10. This assumption is not so restrictive, as we could cast our results in terms of N (where $N = 1, 2, 3, \dots$) rather than k at the expense of an unnecessary increase in length and notation.

11. Our model determines how much each firm is willing to pay for the information and can be augmented to incorporate a stage where the information price is endogenously determined. This, however, would depend critically upon the assumptions we make about the information provision market structure (monopoly versus oligopoly information vendors) as well as the method by which the information is sold (fixed price versus royalties, exclusive versus nonexclusive provision, etc.) and would add considerably to the length and complexity of our paper. We reserve this interesting extension for future research.

The kind of information we have in mind is about consumer characteristics (e.g., gender, age, income group, purchase history). This information, after it has been processed and analyzed, helps the firms segment the consumers into different groups. Firms now can price according to each group's willingness to pay for the different brands. More data about consumers (and/or more sophisticated techniques employed to analyze these data) lead to a finer segmentation. This is consistent with the way most practitioners and empirical researchers view market segments (e.g., Besanko et al., 2003; Rossi et al., 1996). We have made two simplifying assumptions that nevertheless are necessary in order to reduce the complexity of the model: (1) the size of all segments are equal;¹² and (2) the distribution is uniform. Moreover, in practice, a firm's strategy regarding customer information consists of at least two main elements: whether to collect detailed information, and, if so, how much to invest in such a process. More firm resources directed toward this goal should result in consumer databases of higher quality. More importantly, the state of the existing technology imposes an exogenous bound on the quality. For tractability, we focus completely on the first strategic element, by implicitly assuming that the existing technology—which is beyond a firm's control—entirely is responsible for the quality of a customer database and that a firm can choose only whether to acquire such a database or not.

The three-stage game we consider unfolds as follows:

- Stage 1: *Information acquisition decisions*. Given information of quality k , firms decide, simultaneously and independently, whether to acquire it or not.
- Stage 2: *Regular pricing decisions*. Firms, simultaneously and independently, choose their regular prices.
- Stage 3: *Promotional pricing decisions*. The firm(s) with information, simultaneously and independently, distribute(s) targeted price promotions (discounts) to the consumer segments.

Our set-up parallels the multistage games that have been examined in the literature (e.g., Banks and Moorthy, 1999; Rao, 1991; Shaffer and Zhang, 1995, 2002; Thisse and Vives, 1988) where firms choose their promotional strategies after they have chosen their regular prices. This assumption is consistent with the common view that a firm's regular price can be adjusted slower than the choice of targeted coupons. In addition, if both decisions are made simultaneously, no pure strategy

12. See Section 5, where we offer a short discussion on the "equal-sized segments" assumption.

equilibrium exists in the subgames where only one firm has information (see also Shaffer and Zhang, 1995, note 11).¹³

In Section 3, we look for a subgame perfect equilibrium of this game.

3. ANALYSIS

We solve the game backwards starting from stage 3 and proceeding to stage 1. We begin by analyzing the four subgames after the firms' information acquisition decisions in stage 1. The first subgame is when neither firm acquires information and they both set their regular prices in stage 2. Stage 3 never is reached in this case. The second subgame occurs after both firms have acquired information. In stage 2, they both choose their regular prices, and promotions take place in stage 3. Finally, the third and fourth subgames emerge when only one firm possesses information. Both firms set their regular prices in stage 2, and the firm with the information, after it observes the regular price of its rival, offers discounts in stage 3. In Shaffer and Zhang (2002), firms have an incentive to choose a regular price in order to shelter their loyal customers from competitive poaching.¹⁴ This *sheltering* role of the regular price is absent in our model. This is because in our model the firms incur the information cost (if any) in stage 1, and when they choose their promotional strategies this cost is irrelevant. Hence, a firm with information will target each and every consumer regardless of the other firm's regular price. This is not the case in Shaffer and Zhang (2002), where the targeting cost is not sunk when the firms make their targeting decisions. In this case, the targeting benefits should be compared against the targeting costs in order for a firm to decide to which groups of customers to offer promotions. In light of the previous discussion, it is equivalent (and reduces the notational burden) to assume that a firm with information does not choose a regular price in stage 2. Put differently, in the asymmetric subgame, the firm with no information is the price leader (e.g., Thisse and Vives, 1988, p. 129). After having solved all four subgames, we proceed to stage 1 where the firms choose whether to acquire information or not.

3.1 PRICING DECISIONS (STAGES 2 AND 3)

In this subsection, we solve for the equilibrium in each of the four subgames.

13. Proof is available upon request.

14. Loyal to firm 1 are the consumers who prefer firm 1's product when prices are equal, i.e., those in $[0, 1/2]$. The remaining consumers are loyal to firm 2. Customer poaching is a situation where a firm sells to some of its rivals' loyal customers.

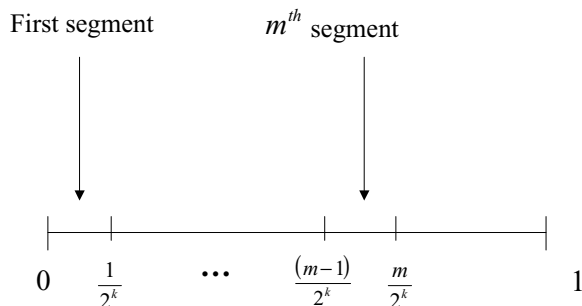


FIGURE 1. PARTITION OF THE UNIT INTERVAL

3.1.1 SUBGAME 1: NEITHER FIRM HAS INFORMATION (NI, NI)

This is the standard Hotelling model under a uniform price. The firms choose their regular prices in stage 2. The demand of each firm's product is given by

$$d_1 = \frac{p_2 - p_1 + t}{2t} \quad \text{and} \quad d_2 = \frac{p_1 - p_2 + t}{2t}.$$

It can be shown easily that in equilibrium the regular prices and profits are $p_1 = p_2 = t$, and

$$\pi_1^{NI,NI} = \pi_2^{NI,NI} = \frac{t}{2}. \quad (1)$$

3.1.2 SUBGAME 2: BOTH FIRMS HAVE INFORMATION (I, I)

Since both firms have consumer information, they know in which of the $N = 2^k$ segments each consumer is located, and therefore they are able to charge different prices for different segments. The interval $[0, 1]$ is divided equally into 2^k segments, each one having length equal to $1/2^k$. Segment m can be expressed as the interval $[(m-1)/2^k, m/2^k]$, where m is an integer between 1 and 2^k (see Figure 1).

In segment m , firms 1 and 2 charge prices p_{1m} and p_{2m} , and the demands of their products are

$$d_{1m} = \frac{p_{2m} - p_{1m} + t}{2t} - \frac{m-1}{2^k} \quad \text{and} \quad d_{2m} = \frac{m}{2^k} - \frac{p_{2m} - p_{1m} + t}{2t},$$

with d_{1m} and d_{2m} in $[0, 1/2^k]$. Their profits are¹⁵

$$\pi_{1m}(p_{1m}, p_{2m}) = p_{1m}d_{1m}, \quad \text{and} \quad \pi_{2m}(p_{1m}, p_{2m}) = p_{2m}d_{2m}.$$

15. For the remainder of the paper, it goes without saying that the segment demands (d_{1m}, d_{2m}) are always in the closed interval $[0, 1/2^k]$.

Firm i 's problem is

$$\max_{p_{im} \geq 0} \pi_{im}(p_{1m}, p_{2m}), \quad \text{for each } m, m = 1, \dots, 2^k, \text{ and } i = 1, 2.$$

The ability of both firms to treat each segment independently of the others allows us to solve for the equilibrium in each subinterval separately and then to aggregate over all subintervals to find the equilibrium profits, denoted by $\pi_1^{I,I}(k)$ and $\pi_2^{I,I}(k)$, as a function of the information quality. Proposition 1 summarizes the solution to the previous problem.

PROPOSITION 1: *Assume that both firms acquire information. Then, for each k ($k \geq 1$), there exist two thresholds (integers) m_1 and m_2 (with $0 \leq m_1 < m_2 \leq 2^k + 1$), where*

$$m_1 = 2^{(k-1)} - 1 \quad \text{and} \quad m_2 = 2^{(k-1)} + 2,$$

such that

- (i) *(This case is valid only when $m_1 \geq 1$) Firm 1's equilibrium demand is equal to $1/2^k$ in all segments from 1 to m_1 , i.e., firm 1 is a constrained monopolist in these segments. Firm 2's equilibrium demand in these segments is zero. Moreover, firm 1's prices are $p_{1m}^* = t(2^k - 2m)/2^k$, while firm 2 sets $p_{2m}^* = 0, m = 1, \dots, m_1$.*
- (ii) *Both firms sell positive quantities in the segments from $m_1 + 1$ to $m_2 - 1$. Moreover, firm 1's prices are $p_{1m}^* = t(2^k - 2m + 4)/(3 \times 2^k)$, and firm 2's prices are $p_{2m}^* = t(2m - 2^k + 2)/(3 \times 2^k), m = m_1 + 1, \dots, m_2 - 1$.*
- (iii) *(This case is valid only when $m_2 \leq 2^k$) Firm 2's equilibrium demand is equal to $1/2^k$ in all segments from m_2 to 2^k , i.e., firm 2 is a constrained monopolist in these segments. Firm 1's equilibrium demand in these segments is zero. Moreover, firm 2's prices are $p_{2m}^* = t(2m - 2^k - 2)/2^k$, while firm 1 sets $p_{1m}^* = 0, m = m_2, \dots, 2^k$.*

Finally, the equilibrium profits of each firm as a function of k are

$$\pi_i^{I,I}(k) = \frac{t(9 - 18 \times 2^{-k} + 40 \times 4^{-k})}{36}, \quad i = 1, 2. \tag{2}$$

Proof. See the Appendix. □

The equilibrium is symmetric. Two-way brand switching—where each firm poaches some of the other firm's loyal customers—occurs in equilibrium always in the middle two consumer segments, i.e., in segments $2^{(k-1)}$ and $2^{(k-1)} + 1$. Consumers located on the left of these two segments buy exclusively from firm 1 and those on the right from firm 2. As $k \rightarrow \infty$ brand switching vanishes.¹⁶

16. Our equilibrium (in this subgame) becomes equivalent to that in Shaffer and Zhang (2000) if we set $k = 1$ (i.e., two identifiable consumer segments) in our model and

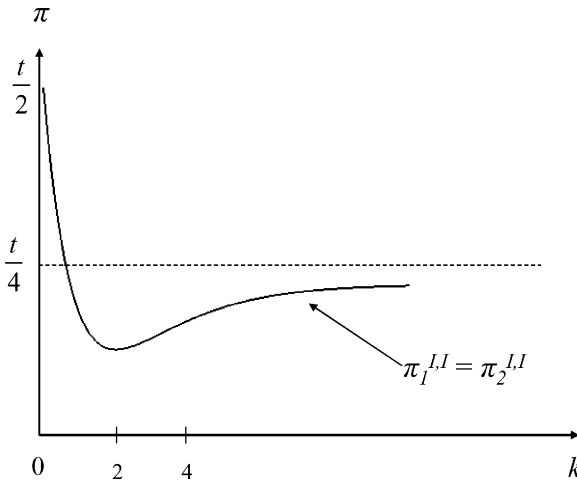


FIGURE 2. PROFITS WHEN BOTH FIRMS HAVE INFORMATION

NUMERICAL EXAMPLE 1

Suppose $k = 3$, i.e., $N = 8$. Then $m_1 = 3$ and $m_2 = 6$, implying that firm 1 and 2 are constrained monopolists in the segments 1, 2, 3 and 6, 7, 8, respectively. In segments 4 and 5 both firms sell positive quantities. The prices that firm 1 charges, starting from segment 1, are $p_{11}^* = 3t/4$, $p_{12}^* = t/2$, $p_{13}^* = t/4$, $p_{14}^* = t/6$, $p_{15}^* = t/12$, and $p_{16}^* = p_{17}^* = p_{18}^* = 0$. Firm 2's prices are symmetric with the highest price in segment 8 and a price equal to zero in segment 1. The equilibrium profits are $\pi_1^{I,I} = \pi_2^{I,I} = .2049t$.

The equilibrium profits exhibit a U-shape as a function of k and are always below $t/2$, the nondiscriminatory profits. As $k \rightarrow \infty$, profits converge to $t/4$, which coincides with the equilibrium profits under perfect price discrimination (see Figure 2).

The intuition behind the nonmonotonicity result will be understood best if we explore the movements of the reaction functions as the quality improves (see Figure 3). Let us begin by assuming that no information is available, i.e., $k = 0$. Firm 1's reaction function is $p_1 = p_2/2 + t/2$, while firm 2's reaction function (after having solved for p_1) is $p_1 = 2p_2 - t$. Both reaction functions are increasing, and they intersect at the symmetric equilibrium price vector (t, t) . The firms, through their pricing strategies, try to strike an optimal balance between

$\theta = 1/2$, $\ell_\alpha = \ell_\beta = t$ and $c = 0$ in theirs [e.g., compare the equilibrium prices and profits after Lemma 2, p. 409 in Shaffer and Zhang (2000) with our results in Proposition 1]. It also becomes equivalent to the equilibrium derived in Chen (1997, p. 883, eq. 1) if we set $\theta = t$ and $c = 0$ in his model and $k = 1$ in ours.

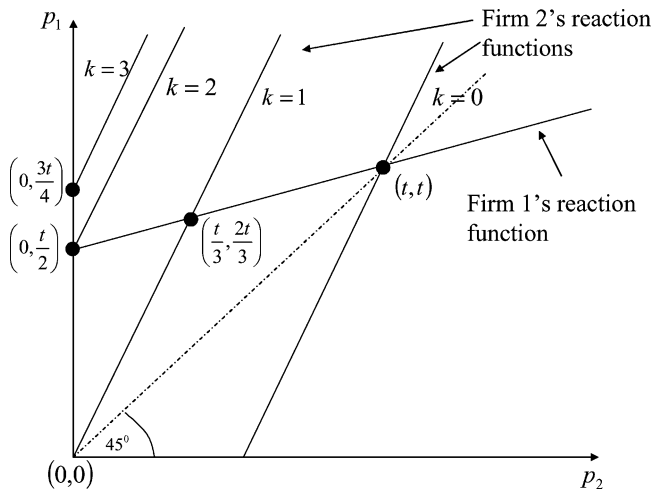


FIGURE 3. REACTION FUNCTIONS AS k INCREASES

gaining (losing) marginal consumers and losing (gaining) inframarginal rents. Now suppose that we move to the next information refinement, i.e., $k = 1$, and let us look at the first segment, i.e., $m = 1$. It can be seen easily that the reaction function of firm 1 remains unchanged but that of firm 2 becomes $p_1 = 2p_2$; that is, it shifts to the left. The complete separation between markets allows firm 2 to charge two different prices, which makes it less concerned about sacrificing its inframarginal rents when it pursues a more aggressive pricing strategy in firm 1's own turf in an attempt to poach some of firm 1's loyal customers (the same is true for firm 1 in firm 2's territory). Since the reaction functions are upward sloping, firm 1 reacts by lowering its price to induce the customers to stay, resulting in lower profits for both firms. When $k = 2$, firm 2's reaction function becomes $p_1 = 2p_2 + t/2$. Firm 2, by applying the same logic, becomes even more aggressive in the first segment, and prices fall further together with both firms' profits. For this k , firm 2's equilibrium price is zero (marginal cost) and sells to no consumer in the first segment, while firm 1 chooses a price equal to $t/2$.

From this particular point on the prices firm 1 charges start to increase with k since it is now quite clear that firm 2 cannot attract any customer from the first segment. When $k = 3$, the equilibrium price vector is $(0, 3t/4)$. So, if we look at the consumers who are located very close to firm 1, they initially face a decreasing sequence of prices, but after a certain threshold the prices they pay increase with k , resulting in profits that are U-shaped. This line of reasoning can be extended to

any location on the interval (of course, in the interior segments firm 1's reaction function changes as well), and the aggregation of profits over all consumers yields a total profit function that also exhibits a U-shape.¹⁷

To sum up, there are two forces at work: the intensified competition effect and the surplus extraction effect. When both firms sell positive quantities in a given segment of consumers, an information refinement intensifies the competition, and prices fall. This occurs for low levels of information quality. For high precision, this fighting over consumers ceases as it now becomes more apparent which brand the consumers in a given segment prefer, and the other firm cannot attract them even if it offers its product at marginal cost. Any further information improvements allow the firms to extract more surplus by raising their prices.

3.1.3 SUBGAMES 3 AND 4: ONLY ONE FIRM HAS INFORMATION (I, NI) OR (NI, I)

Due to symmetry, let us assume that firm 1 is the firm who has information. Firm 2 chooses its regular price p_2 in stage 2, and firm 1 chooses its promotional prices p_{1m} , $m = 1, \dots, 2^k$ in stage 3. We first solve firm 1's problem. Its demand in each segment is

$$d_{1m} = \frac{p_2 - p_{1m} + t}{2t} - \frac{m-1}{2^k}, \quad \text{for } m = 1, \dots, 2^k.$$

Given p_2 firm 1 chooses the depth of the discount that it offers to maximize

$$\pi_1^{I,NI} = \sum_{m=1}^{2^k} p_{1m} d_{1m}.$$

Let $p_{1m}^*(k, p_2)$, $m = 1, \dots, 2^k$ denote the solution to the firm's maximization problem. Now let us turn to firm 2's problem. Its demand in each segment is

$$d_{2m} = \frac{m}{2^k} - \frac{p_2 - p_{1m}^*(k, p_2) + t}{2t}, \quad \text{for } m = 1, \dots, 2^k.$$

17. The modeling implications of a departure from the uniform distribution deserve some discussion. The nonmonotonicity of the equilibrium profits with respect to k (information quality), when both firms have acquired information, holds for a wide class of distribution functions. First, we were able to show that the general structure we demonstrated in Proposition 1 is true for any distribution that satisfies the monotone hazard rate property plus one more (not very restrictive) condition. Specifically, both firms sell positive quantities only in the middle segments, and the mass of these segments vanishes as k approaches infinity. Second, based on the result just determined we proved that the equilibrium profits initially decrease and eventually increase (with k), although we were unable to characterize the behavior of the profits for intermediate values of k . (Proof is available upon request.) Finally, without the uniform assumption, it was not possible to compare the profit configurations clearly among the four different subgames.

Given the reaction function of firm 1, firm 2 chooses its regular price p_2 to maximize

$$\pi_2^{I,NI} = p_2 \sum_{m=1}^{2^k} d_{2m}.$$

Let $\pi_1^{I,NI}(k)$ and $\pi_2^{I,NI}(k)$ denote the equilibrium profits, when only firm 1 has information, as a function of the information quality. Proposition 2 summarizes the properties of the solution. In the proof of this proposition we no longer can solve for the equilibrium in each segment separately, as we did in Proposition 1, because due to its inability to charge more than one price firm 2 cannot treat the consumer segments independently. This *unbalanced* distribution of information between the two firms adds to the difficulty and length of the proof significantly. Nonetheless, we were able to obtain closed-form solutions and to characterize the problem to its fullest extent.

PROPOSITION 2: *Assume that only firm 1 has acquired information. Then, for each k ($k \geq 1$), there exist two thresholds (integers) m_1 and m_2 (with $0 \leq m_1 < m_2 \leq 2^k + 1$), where*

$$m_1 = 3 \times 2^{(k-2)} - 1 \quad \text{and} \quad m_2 = 3 \times 2^{(k-2)} + 2, \quad \text{for } k \geq 2, \quad \text{and} \\ m_1 = 0, \quad m_2 = 3 \quad \text{for } k = 1,$$

such that

- (i) Firm 2's regular price is $p_2 = t(1/2 + 2^{-(k+1)})$.
- (ii) (This case is valid only when $m_1 \geq 1$) Firm 1's equilibrium demand is equal to $1/2^k$ in all segments from 1 to m_1 ; i.e., firm 1 is a constrained monopolist in these segments. Firm 2's equilibrium demand in these segments is zero. Moreover, firm 1's prices are: $p_{1m}^* = t(3/2 + 2^{-(k+1)} - 2^{(1-k)m})$, $m = 1, \dots, m_1$.
- (iii) Both firms sell positive quantities in the segments from $m_1 + 1$ to $m_2 - 1$. Moreover, firm 1's prices are $p_{1m}^* = t(3/4 + (5/4) \times 2^{-k} - 2^{-k}m)$, $m = m_1 + 1, \dots, m_2 - 1$.
- (iv) (This case is valid only when $m_2 \leq 2^k$) Firm 2's equilibrium demand is equal to $1/2^k$ in all segments from m_2 to 2^k ; i.e., firm 2 is a constrained monopolist in these segments. Firm 1's equilibrium demand and prices in these segments are zero, i.e., $p_{1m}^* = 0$, $m = m_2, \dots, 2^k$.

Finally, the equilibrium profits of each firm as a function of k are

$$\pi_1^{I,NI}(k) = \frac{t(9 - 6 \times 2^{-k} + 5 \times 4^{-k})}{16} \quad \text{and} \\ \pi_2^{I,NI}(k) = \frac{t(1 + 2 \times 2^{-k} + 4^{-k})}{8}. \tag{3}$$

Proof. See the Appendix. □

Here, unlike the case where both firms have information, two-way brand switching occurs in equilibrium only when $k = 1$, i.e., two identifiable consumer groups. For any $k \geq 2$, the market experiences a one-way poaching where only firm 1 steals some of firm 2's loyal customers, a situation that persists as $k \rightarrow \infty$.

NUMERICAL EXAMPLE 2

Suppose $k = 3$, i.e., $N = 8$. Then $m_1 = 5$ and $m_2 = 8$, implying that firm 1 and 2 are constrained monopolists in the segments 1, 2, 3, 4, 5, and 8, respectively. In segments 6 and 7 both firms sell positive quantities. Firm 2's regular price is $p_2 = .5625t$. The prices that firm 1 charges, starting from segment 1, are $p_{11}^* = 1.3125t$, $p_{12}^* = 1.0625t$, $p_{13}^* = .8125t$, $p_{14}^* = .5625t$, $p_{15}^* = .3125t$, $p_{16}^* = .15625t$, $p_{17}^* = .03125t$, and $p_{18}^* = 0$. The equilibrium profits are $\pi_1^{I,NI} = .5205t$ and $\pi_2^{I,NI} = .1582t$.

The profits of both firms are the same ($\pi_1 = \pi_2 = t/2$) when $k = 0$ (no information is available). Then, as k increases, firms 2's profits monotonically decrease, and they approach $t/8$ as the information tends to become perfect (i.e., $k \rightarrow \infty$). Firm 1's profits, however, are nonmonotonic in k . For low information quality (i.e., only for $k = 1, 2$), firm 1's profits with information are lower (i.e., less than $t/2$) than the ones without information, even with a zero cost of information. This is because firm 2 is aware of the fact that firm 1 has committed credibly to price discriminate, which forces firm 2 to follow a more defensive pricing strategy by lowering its regular price significantly. On the other hand, the quality of information is initially quite low, and therefore firm 1 cannot take full advantage of the information benefits. When the precision increases, the surplus extraction effect becomes more dominant, and firm 1's profits increase and they approach $9t/16$ as $k \rightarrow \infty$ (see Figure 4).

3.2 INFORMATION ACQUISITION DECISIONS (STAGE 1)

The game played between the two firms in the first stage can be summarized in Table I.

TABLE I.
(STAGE 1) FIRMS' PAYOFFS

1\2	NI	I
NI	$(\frac{t}{2}, \frac{t}{2})$	$(\frac{t(1+2 \times 2^{-k} + 4^{-k})}{8}, \frac{t(9-6 \times 2^{-k} + 5 \times 4^{-k})}{16})$
I	$(\frac{t(9-6 \times 2^{-k} + 5 \times 4^{-k})}{16}, \frac{t(1+2 \times 2^{-k} + 4^{-k})}{8})$	$(\frac{t(9-18 \times 2^{-k} + 40 \times 4^{-k})}{36}, \frac{t(9-18 \times 2^{-k} + 40 \times 4^{-k})}{36})$

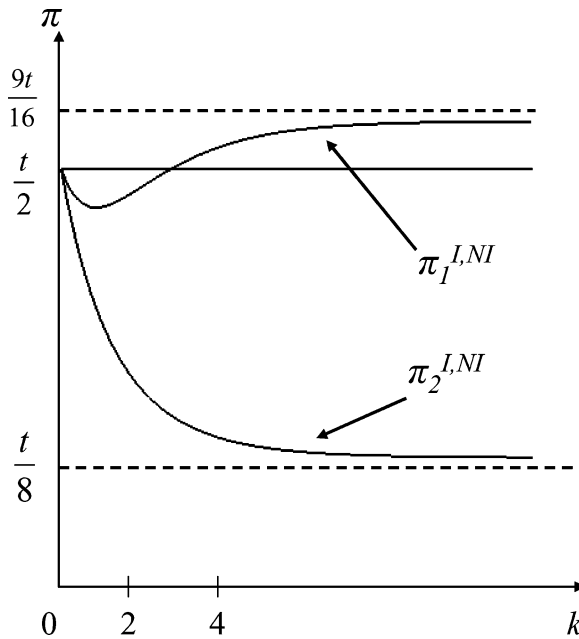


FIGURE 4. PROFITS WHEN ONLY FIRM 1 HAS INFORMATION

The profits in Table I have been taken from (1), (2), and (3). Our 2×2 game, as it is captured in Table I, has the 2×2 game studied by Thisse and Vives (1988) as its limit point (to see this let $k \rightarrow \infty$ in our game, and set $c = 0$ in Thisse and Vives, 1988, table 1, p. 131). We assume that the information price is sufficiently low so that it essentially does not influence the firms' acquisition decisions.¹⁸ Proposition 3 summarizes the equilibrium in the game.

PROPOSITION 3: *When $k < 3$, no information (NI) is each firm's (strictly) dominant strategy, while for $k \geq 3$, acquiring information (I) becomes the (strictly) dominant strategy, and the game is a prisoners' dilemma.*

The proof of Proposition 3 can be seen best by combining Figures 2 and 4 in a single graph (see Figure 5).

We plot the equilibrium profits of firm 1 as a function of k for each one of the four subgames. The graph clearly shows that NI is firm 1's strictly dominant strategy when $k < 3$ (i.e., $k = 1$ or 2), while for $k \geq 3$ I is strictly dominant. Due to symmetry, a similar graph and conclusion

18. To include an exogenous information cost (not just a sufficiently low one) is a straightforward exercise and does not add much to our understanding of the problem.

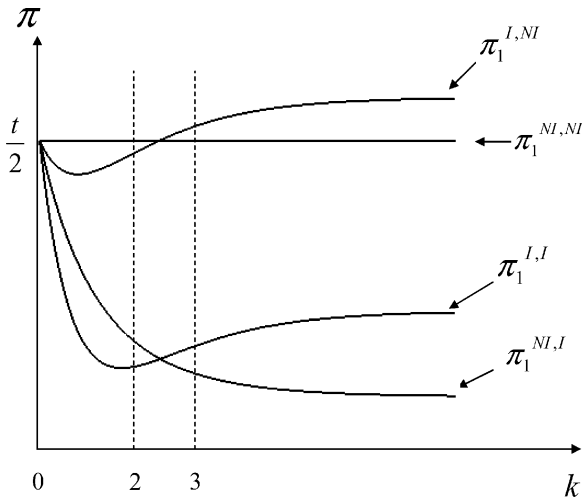


FIGURE 5. STAGE 1 PROFITS OF FIRM 1

can be drawn for firm 2. Hence, for $k < 3$, the unique equilibrium is for neither firm to acquire consumer information, whereas for $k \geq 3$ the unique equilibrium is for both firms to acquire information. There does not exist an asymmetric equilibrium, where one firm acquires information and the other does not. Note that equilibrium profits when $k \geq 3$ are always less than $t/2$, the uniform-price profits.

Consumer information generates two effects for the firm that acquires it. First, it allows the firm to extract more surplus from all consumers, and second it forces the firm's rival to price more defensively in response. When the quality of information is low ($k < 3$, or 8 consumer segments) the second effect outweighs the first, and each firm finds it not worthwhile to trigger its rival's defensive response. This is achieved by credibly committing, through the nonacquisition of information, not to charge more than one price. When the information becomes more refined, the first effect is stronger, and as a consequence the firms cannot resist the temptation any longer. They both acquire information, which leads to a prisoners' dilemma.

4. DISCUSSION OF THE MAIN RESULTS

In this section, we build upon the results derived in Section 3 to evaluate the implications of information improvements for the firms' profits and consumer and social welfare.

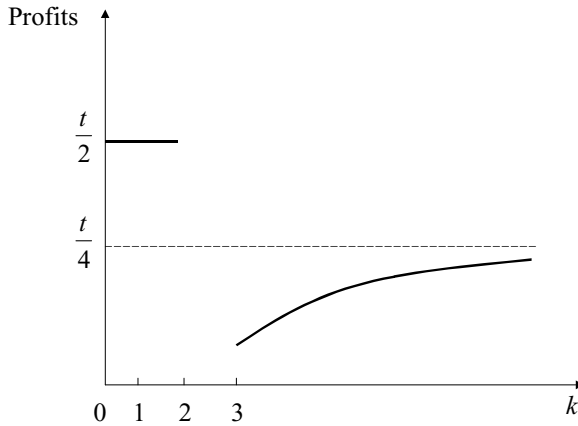


FIGURE 6. EQUILIBRIUM PROFITS IN THE THREE-STAGE GAME

4.1 IMPLICATIONS FOR FIRMS

From Proposition 3, firm *i*'s equilibrium profits in the three-stage game are

$$\pi_i(k) = \begin{cases} \frac{t}{2}, & \text{if } k < 3 \\ \frac{t(9 - 18 \times 2^{-k} + 40 \times 4^{-k})}{36}, & \text{if } k \geq 3. \end{cases}$$

Profits exhibit a U-shape (with some abuse of the term) as a function of *k* (see Figure 6). Both firms' relative position improves with *k* for *k* ≥ 3. Despite the fact that a commitment device not to deviate from a uniform price (by not acquiring information) technologically is feasible, firms do not find it in their best interest to utilize such a restraint unilaterally if its quality is high for *k* ≥ 3. This complements the findings in Corts (1998, p. 320, proposition 7) and Thisse and Vives (1988, p. 130, proposition 2), who respectively show that when firms can discriminate only between two consumer groups (*k* = 1 in our model), such unilateral commitments in stage 1 do constitute an equilibrium, whereas under perfect discrimination (*k* = ∞ in our model) they do not and the game is a prisoners' dilemma. Our analysis shows that (1) the unique threshold of information quality where this policy switching occurs is when the firms have the ability to identify eight consumer groups; and (2) the game is a prisoners' dilemma for all *k*'s greater than 3 (eight groups) and not only for *k*'s arbitrarily large.¹⁹

19. With asymmetric firms and perfect price discrimination, Shaffer and Zhang (2002) demonstrate that the game need not be a prisoners' dilemma.

In practice, firms may make an attempt to soften the intensity of competition through various commitments such as the adoption of everyday low pricing or a no-haggle policy (see Corts, 1998, Section 4). Our game can be modified slightly in order to provide a more complete view of this issue. Suppose that firms already possess consumer information of quality k . In stage 1, they decide whether to commit not to use it by unilaterally announcing publicly the adoption of one of the aforementioned policies; stages 2 and 3 are the same as in our game. Then, it follows easily from our analysis so far that such commitments will prevail in equilibrium only for low information quality ($k < 3$). In any other case firms simply find it in their best interest to utilize their information and to price discriminate.

4.2 IMPLICATIONS FOR CONSUMERS

The welfare implications for the consumers are as follows. Ignoring for the moment the transportation cost, consumers become better off compared to the no-discrimination case, as the prices each firm charges are uniformly below the nondiscriminatory price t for any k . To see this consider for example firm 1 (the same holds for firm 2). When firm 1 is a constrained monopolist it charges a price equal to $t(2^k - 2m)/2^k \leq t$, $m = 1, \dots, m_1$ (Proposition 1). A similar result holds in the segments where the two firms sell positive quantities. The reason is that markets are treated asymmetrically by the two firms. Firm 1's strongest market (i.e., the group of consumers closest to firm 1) is firm 2's weakest market and so on. When discrimination is practiced against two consumer groups, Corts (1998, p. 311) demonstrated that this asymmetric treatment of the markets (termed *best-response asymmetry*) is a necessary condition for price discrimination to yield unambiguous price effects.

The new question this paper addresses is how do the average price and consumer welfare evolve as k increases?

The average price (AVP) and transportation cost ($AVTC$) are²⁰

$$AVP(k) = \frac{t(9 - 18 \times 2^{-k} + 40 \times 4^{-k})}{18},$$

and

$$AVTC(k) = \frac{t(9 + 8 \times 4^{-k})}{36}.$$

Note that the average price exhibits a U-shape pattern similar to that of the profits in Figure 2. The average consumer surplus is

20. The file with the calculations is available upon request.

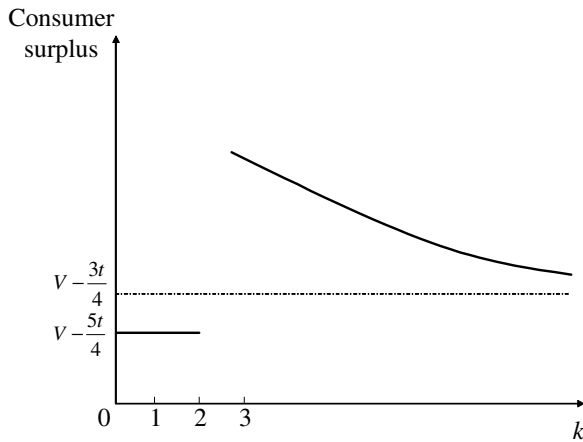


FIGURE 7. CONSUMER SURPLUS IN THE THREE-STAGE GAME

$V - AVP(k) - AVTC(k)$, which is

$$AVCS(k) = V - \frac{t(27 - 36 \times 2^{-k} + 88 \times 4^{-k})}{36}.$$

Moreover, when no information is used (i.e., *NI, NI*) the average consumer surplus is $V - 5t/4$, which can be seen easily that it is always less than that when information is used (i.e., *I, I*) for any k . Therefore, the average consumer surplus for the entire game is

$$AVCS(k) = \begin{cases} V - \frac{5t}{4}, & \text{if } k < 3 \\ V - \frac{t(27 - 36 \times 2^{-k} + 88 \times 4^{-k})}{36}, & \text{if } k \geq 3. \end{cases}$$

Consumer surplus exhibits an inverse U-shape (again, with some abuse of the term) as a function of k , implying that moderate levels of information yield the highest surplus. Therefore, consumers are better off when firms use information than when no information is used, but given that information is used there is an optimal (for consumer welfare) level that is finite. After the peak of consumer welfare, some consumers pay higher prices as information quality increases (see Figure 7), and consumer welfare approaches asymptotically $V - 3t/4$.²¹

21. Social welfare comparisons in a covered market with inelastic demands are not very interesting. One can note easily that brand switching creates a deadweight loss to the society as, for example, in Chen (1997), Fudenberg and Tirole (2000), and Shaffer and Zhang (1995). As information improves the mass of consumers that switches brands shrinks to zero, and social welfare increases.

5. CONCLUDING REMARKS

We propose and analyze a spatial price discrimination model that encompasses two-group and perfect discrimination as special cases. Our model offers a more comprehensive view of the firms' incentives to engage in price discrimination, as well as of the transition of profits and welfare as IT advances. In particular, we evaluate the role of an improving information quality on (1) the firms' incentives to acquire it; (2) the equilibrium profits; and (3) consumer and social welfare. We do so by analyzing a three-stage game. In the first stage the firms make their information acquisition decisions; in the second stage they choose their regular prices; and in the third stage they choose their promotional prices. The information allows the sellers to group the customers into different segments according to their brand preferences and to charge each segment a different price. An information improvement increases the number of identifiable segments. Our main results can be summarized as follows:

- (1) *Information acquisition decisions*: When the consumer segmentation is relatively coarse, it is each firm's dominant strategy to credibly commit not to price discriminate. The game is not a prisoners' dilemma. After a unique threshold of information precision acquiring information with the intent to price discriminate becomes a dominant strategy resulting in a prisoners' dilemma.
- (2) *Equilibrium profits*: Equilibrium profits are U-shaped with respect to information improvements and always lower than the ones under a uniform price. Thus, the availability of information hurts the firms' profits, but conditional on its availability, a refinement initially leads to lower profits—but after a threshold profits increase.
- (3) *Consumer welfare*: Consumer welfare exhibits an inverse U-shape as a function of the information quality, and it is always higher than the one under no price discrimination. Moderate information quality is the most beneficial for the consumers.

Chen et al. (2001) predict that the equilibrium profits are inverse U-shaped as a function of the information accuracy. In their duopoly model there are three groups (types) of consumers: loyal to each firm consumers who do not compare prices and switchers who buy from the cheaper firm. Firms receive imperfect signals about the loyalty of each customer. At low levels of targetability profits increase as the accuracy improves. This is due to two factors. First, the firms are able to extract more surplus from their loyal customers, as now they can identify them better. Second, the competition between the two firms for the switchers is very soft since they cannot be separated clearly from the

loyal customers. As the level of targetability improves profits eventually decrease because the consumers can be segmented more accurately, and the ensuing intense competition for the switchers outweighs the surplus extraction benefits.

It is useful to elaborate more on the differences between our predictions and the predictions in Chen et al. (2001). In Chen et al., the information comes in the form of a noisy signal and therefore lacks that kind of certainty (present in our framework) about who anyone is. This observation is crucial and is responsible for generating the different results. In our model, when information is coarse the intensified competition effect dominates the surplus extraction effect. A firm knows that a particular customer prefers for certain the rival firm's product. Therefore, the firm does not sacrifice a significant portion of its revenue by offering the product to this consumer at a low price. The rival firm, in response, lowers its price as well in an attempt to retain the consumer. At the same time, the information is not fine enough to allow firms to extract surplus, and, therefore, profits initially decrease. As the partition becomes more refined, the fighting between rival firms over consumers subsides, and surplus extraction starts to dominate. At this point profits begin to ascend (U-shape). In Chen et al. (2001), the relative strength of these two effects is the opposite as a function of the signal precision. Suppose a firm obtains a signal that a particular customer prefers the rival brand. This signal is informative but nevertheless is noisy. A firm will price cautiously since this consumer may be its own. As a result the other firm does not have to lower its price too much. Since the signal contains some information and competition is not so intense, profits initially increase. As the quality of the signal improves, it becomes evident who each customer is, and competition becomes fiercer, resulting in lower profits (inverse U-shape).

Consider the following example that further highlights these differences.²² Suppose there are two competing automobile manufacturers, one American and one European. There are two different types of information about consumers' relative strength of preferences for the two brands. The first one reveals the purchase history of consumers. If a consumer is a lifetime purchaser of American cars, then the two manufacturers can use this information to classify this consumer into a group loyal to American cars. As a result, the European manufacturer can price to this particular group really low, being certain that he is not hurting himself. This partition of consumers into distinct groups is more consistent with our definition of information. On the other hand, suppose the information is about demographics and in particular

22. We thank an anonymous referee for suggesting this example to us.

about where each consumer resides. Suppose a consumer lives in a neighborhood, where 50% of the residents drive American cars and 20% drive European cars. This information is not enough to make the European manufacturer price too aggressively, because, after all, the specific customer has a good chance of being loyal to the European models. This is more consistent with the definition of information in Chen et al. (2001).

The inverse U-shape result is also consistent with a common value auction model with a continuum of types. In such a setting, Matthews (1984, pp. 191–192) showed that the bidders' profits are inverse U-shaped as a function of the signal precision. Our conjecture is that if we view the two firms as the bidders who enter into a bidding competition for each individual consumer's business, then equilibrium profits may follow an inverse U-shape. Although the two extreme cases of "zero" and perfect information should yield the same profits and welfare irrespective of the approach taken (i.e., partition, as in our paper or signal), the intermediate cases of imperfect accuracy seem to produce radically different outcomes depending on the specific way an information improvement is entering the model. This has far-reaching implications and raises a number of interesting questions, such as (1) how should an information vendor "package" his information? and (2) should regulators promote one "type" of information and ban another? The present paper is a step in this direction, and more research needs to be done to fully understand the impact of information advancements on profits and welfare.

Our modeling framework has a number of limitations. Some limitations are inherent to the Hotelling model in general (e.g., symmetric demands, uniform distribution), while others are more specific to our model (e.g., equal-sized segments). We have discussed already the consequences of relaxing the uniform distribution assumption in note 17. Following, we list the remaining assumptions we think are most likely to be responsible for the results we obtained.

5.1 EQUAL SIZED SEGMENTS

The assumption that the information partitions the unit interval in equally sized segments may be restrictive. Suppose for example that firms obtain consumer information exclusively through past transactions with them.²³ To simplify the exposition assume that firms and consumers use a zero discount factor (myopic) and that firms cannot commit not to use the information they already possess. At the beginning

23. We thank a coeditor for urging us to think along these lines.

of period 1, firms have no information about the consumers, besides the fact that they are uniformly distributed. Each seller charges one price. The first half of the consumers buy from firm 1, and the rest from firm 2. Therefore, at the end of period 1, firms are able to segment the consumers into two equally sized segments, $[0, 1/2]$ and $[1/2, 1]$. At the beginning of period 2 each firm charges two prices, and the consumers at the end of period 2, are segmented into four segments. The segments now are not equally sized. The end segments are larger than the middle segments. It can be shown easily that the end segments are $[0, 1/3]$ and $[2/3, 1]$, while the middle segments are $[1/3, 1/2]$ and $[1/2, 2/3]$. It also can be shown that the end segments will converge (monotonically) to $[0, 1/4]$ and $[3/4, 1]$ as time goes to infinity. Nevertheless, the middle segments are more refined. The firms never really learn enough about their most loyal customers, since these consumers never switch brands. This lessens the surplus extraction effect. Our conjecture is that the U-shape result may become less pronounced or even may disappear. Nevertheless, in reality—as we have pointed out in Section 2—firms do not have to rely exclusively on past transactions to obtain information about their loyal customers. They can acquire customer specific information through other sources as well.

5.2 SYMMETRIC DEMANDS

If demands were asymmetric (e.g., one firm had a larger customer base than the other, all else equal), then our results would not necessarily hold. Shaffer and Zhang (2002) have demonstrated, in a perfect price discrimination context, that it is the larger firm who benefits from consumer information at the expense of its smaller rival. This result is in sharp contrast with the prisoners' dilemma we have identified in our paper. An interesting research question is to investigate the incentives of asymmetric firms to invest in imperfect customer information. In future work, we intend to build on the techniques developed in the present paper to tackle this question.

APPENDIX

Proof of Proposition 1. We conjecture the following structure. There exist two integers m_1 and m_2 with $0 \leq m_1 < m_2 \leq 2^k + 1$, such that (1) (*left segments*) firm 1 is a constrained monopolist in all segments from 1 to m_1 (if $m_1 = 0$, then firm 1 is never a constrained monopolist); (2) (*middle segments*) in all segments from $m_1 + 1$ to $m_2 - 1$ the two firms sell positive quantities; and (3) (*right segments*) in all segments from m_2 to 2^k firm 2 is a constrained monopolist (again, if $m_2 = 2^k + 1$ firm 2 is never a

constrained monopolist). Next, we set out to prove that this structure indeed holds.

- (1) Both firms charge strictly positive prices (middle segments). Ignoring the nonnegativity constraints and setting $\partial\pi_{im}/\partial p_{im} = 0$, $i = 1, 2$, we obtain the following solutions for the prices:

$$p_{1m} = \frac{t(2^k - 2m + 4)}{3 \times 2^k},$$

and

$$p_{2m} = \frac{t(2m - 2^k + 2)}{3 \times 2^k}.$$

Using these prices we obtain the demands

$$d_{1m} = \frac{-2m + 2^k + 4}{6 \times 2^k},$$

and

$$d_{2m} = \frac{2m - 2^k + 2}{6 \times 2^k}.$$

We can see that d_{1m} is decreasing in m , and that d_{2m} is increasing in m . This means that firm 1 may decide to charge a zero price and to give the entire segment demand to firm 2 for segments that are in firm 2's territory. Analogously, for firm 2 in segments that are in firm 1's territory. For segments in the middle of the interval, both firms charge positive prices. Observe that $d_{2m} = (2m - 2^k + 2)/(6 \times 2^k) \leq 0$ for any $m \leq 2^{k-1} - 1$, and $d_{1m} = (2^k - 2m + 4)/(6 \times 2^k) \leq 0$ for any $m \geq 2^{k-1} + 2$. Now define $m_1(k)$ to be the largest integer that is less than or equal to $2^{k-1} - 1$, and $m_2(k)$ to be the smallest integer that is greater than or equal to $2^{k-1} + 2$. Obviously,

$$m_1 = 2^{(k-1)} - 1 \quad \text{and} \quad m_2 = 2^{(k-1)} + 2.$$

This will be used later in the proof. Hence, for any $m = m_1 + 1, \dots, m_2 - 1$, both firms charge strictly positive prices and have strictly positive segment demands.

- (2) Firm 1 charges strictly positive prices while firm 2 charges a zero price (left segments). Following the previous analysis, this case is valid for $m \leq m_1$. Then $d_{2m} \leq 0$. This implies that $d_{2m} = 0$ and $d_{1m} = 1/2^k$. This further implies that $p_{2m} = 0$, and p_{1m} is the solution to $d_{1m}(p_{2m} = 0) = 1/2^k$, which yields $p_{1m} = t(2^k - 2m)/2^k$.
- (3) Firm 2 charges strictly positive prices while firm 1 charges a zero price (right segments). This case is valid for $m \geq m_2$. This case is

symmetric to case 2. Firm 2's prices in these segments are: $p_{2m} = t(2m - 2^k - 2)/2^k$. Below we summarize the results:

The equilibrium prices and profits are

(i) if $m_1 + 1 \leq m \leq m_2 - 1$ (middle segments), then

$$p_{1m} = \frac{t(2^k - 2m + 4)}{3 \times 2^k} \quad \text{and} \quad p_{2m} = \frac{t(2m - 2^k + 2)}{3 \times 2^k}$$

$$d_{1m} = \frac{-2m + 2^k + 4}{6 \times 2^k} \quad \text{and} \quad d_{2m} = \frac{2m - 2^k + 2}{6 \times 2^k}$$

$$\pi_{1m}^{I,I}(k) = \frac{t(2m - 2^k - 4)^2}{18 \times 4^k} \quad \text{and} \quad \pi_{2m}^{I,I}(k) = \frac{t(2m - 2^k + 2)^2}{18 \times 4^k};$$

(ii) if $m \leq m_1$ (left segments), then

$$p_{1m} = \frac{t(2^k - 2m)}{2^k} \quad \text{and} \quad p_{2m} = 0$$

$$d_{1m} = \frac{1}{2^k} \quad \text{and} \quad d_{2m} = 0$$

$$\pi_{1m}^{I,I}(k) = \frac{t(2^k - 2m)}{4^k} \quad \text{and} \quad \pi_{2m}^{I,I}(k) = 0; \quad \text{and}$$

(iii) if $m \geq m_2$ (right segments),

$$p_{1m} = 0 \quad \text{and} \quad p_{2m} = \frac{t(2m - 2^k - 2)}{2^k}$$

$$d_{1m} = 0 \quad \text{and} \quad d_{2m} = \frac{1}{2^k}$$

$$\pi_{1m}^{I,I}(k) = 0 \quad \text{and} \quad \pi_{2m}^{I,I}(k) = \frac{t(2m - 2^k - 2)}{4^k}.$$

Therefore, firms' profits for each k are

$$\pi_1^{I,I}(k) = \sum_{m=1}^{m_1} \frac{t(2^k - 2m)}{4^k} + \sum_{m=m_1+1}^{m_2-1} \frac{t(2m - 2^k - 4)^2}{18 \times 4^k},$$

$$\pi_2^{I,I}(k) = \sum_{m=m_1+1}^{m_2-1} \frac{t(2m - 2^k + 2)^2}{18 \times 4^k} + \sum_{m=m_2}^{2^k} \frac{t(2m - 2^k - 2)}{4^k}.$$

By performing the summation and using ($m_1 = 2^{(k-1)} - 1$ and $m_2 = 2^{(k-1)} + 2$), we obtain

$$\pi_i^{I,I}(k) = \frac{t(9 - 18 \times 2^{-k} + 40 \times 4^{-k})}{36}, \quad i = 1, 2. \quad \square$$

Proof of Proposition 2. We conjecture the same structure as in the proof of Proposition 1. We break the proof into the following three steps: (1) step 1, where we derive firm 1's reaction functions and its profit function; (2) step 2, where we derive firm 2's profit function, guess the optimal price p_2 , and show that the first-order derivative vanishes at the guessed price; and (3) step 3, where we prove that the guessed price is indeed the unique solution by showing that firm 2's profit function is differentiable (and hence continuous) and strictly concave, which implies that a unique maximum exists and is where the first derivative becomes zero.

- (1) Step 1. Let us begin by analyzing firm 1's problem given the regular price p_2 of the other firm. Firm 1's demand and profit functions in segment m are

$$d_{1m} = \frac{p_2 - p_{1m} + t}{2t} - \frac{m-1}{2^k} \quad \text{and} \quad \pi_{1m}(p_{1m}; p_2, k) = p_{1m}d_{1m}.$$

- (a) Left segments. For each $m = 1, \dots, m_1$ firm 1's demand is $1/2^k$ and the price in each one of these segments is

$$p_{1mL} = p_2 + t - \frac{2tm}{2^k}.$$

For this to be firm 1's best response it must be that

$$\left. \frac{\partial \pi_{1m}}{\partial p_{1m}} \right|_{p_{1mL} = p_2 + t - \frac{2tm}{2^k}} = \frac{2tm + 2t - 2^k p_2 - 2^k t}{2^{k+1}t} \leq 0. \quad (\text{A1})$$

By setting (A1) equal to zero and solving with respect to m , we obtain

$$m^* = \frac{2^k p_2 + 2^k t - 2t}{2t}. \quad (\text{A2})$$

For any $m < m^*$, (A1) is strictly negative. However, m^* is not an integer (except for specific p_2 's). Let $m_1(p_2, k)$ be the largest integer that is less than or equal to m^* . This will be the last segment where firm 1 is a constrained monopolist.

- (b) Right segments. In the right segments firm 1's price (and demand) is zero, i.e., $p_{1mR} = 0$. For this to be a best response it must be that the marginal consumer, who is located at $(m-1)/2^k$, is indifferent between buying from firm 1 at zero price and from firm 2 at p_2 , i.e.,

$$0 + t \frac{(m-1)}{2^k} = p_2 + t \left[1 - \frac{(m-1)}{2^k} \right]. \quad (\text{A3})$$

By solving (A3) with respect to m we obtain

$$m^{**} = \frac{2^k p_2 + 2^k t + 2t}{2t}. \tag{A4}$$

All consumers located to the right of $(m^{**} - 1)/2^k$ buy from firm 2, even when firm 1 charges a zero price. However, m^{**} is not an integer (except for specific p_2 's). Let $m_2(p_2, k)$ be the smallest integer that is greater than or equal to m^{**} . This will be the first segment where firm 2 is a constrained monopolist.

(c) Middle segments. The demands of both firms are strictly positive. Firm 1's best-response function is

$$\frac{\partial \pi_{1m}}{\partial p_{1m}} = 0 \Rightarrow p_{1mM} = \frac{2^k p_2 + 2^k t - 2tm + 2t}{2^{k+1}}.$$

By considering all segments together, firm 1's profit function is

$$\begin{aligned} \pi_1(p_2, k) = & \sum_{m=1}^{m_1(p_2, k)} \frac{1}{2^k} p_{1mL} \\ & + \sum_{m=m_1(p_2, k)+1}^{m_2(p_2, k)-1} \left[\frac{p_2 - p_{1mM} + t}{2t} - \frac{m-1}{2^k} \right] p_{1mM} \\ & + \sum_{m=m_2(p_2, k)}^{2^k} 0. \end{aligned} \tag{A5}$$

(2) Step 2. We continue by studying firm 2's problem. Its demand function in each segment is

$$d_{2m} = \frac{m}{2^k} - \frac{p_2 - p_{1m}(p_2, k) + t}{2t}.$$

Firm 2 chooses p_2 to maximize

$$\begin{aligned} \pi_2(p_2, k) = & \left[\sum_{m=1}^{m_1(p_2, k)} 0 + \sum_{m=m_1(p_2, k)+1}^{m_2(p_2, k)-1} \frac{m}{2^k} \right. \\ & \left. - \frac{p_2 - p_{1mM}(p_2, k) + t}{2t} + \sum_{m=m_2(p_2, k)}^{2^k} \frac{1}{2^k} \right] p_2. \end{aligned} \tag{A6}$$

We cannot solve for the optimal p_2 directly because we cannot obtain a closed-form expression for $\pi_2(p_2, k)$. This is due to the fact that we do not have closed-form expressions for $m_1(p_2, k)$ and $m_2(p_2, k)$. Recall that these two thresholds were found by integerizing (A2)

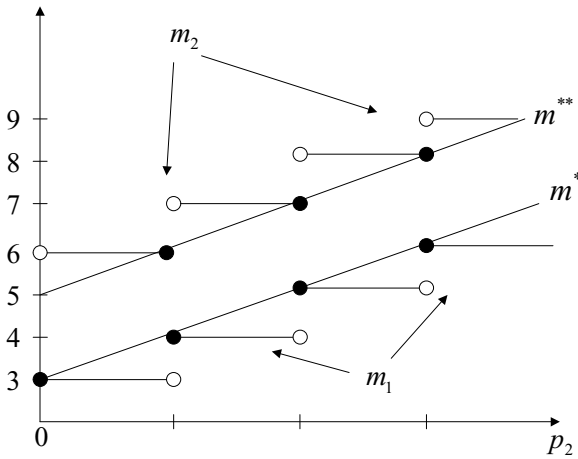


FIGURE A1. THE TWO STEP FUNCTIONS ($k = 3$ AND $t = 1$)

and (A4), and they are actually step functions (as a function of p_2) that complicates the analysis [see Figure A1 where we have plotted m^* , m^{**} (the straight lines) and m_1, m_2 (the step functions)]. Nevertheless, we were able to circumvent this problem and to obtain a closed-form solution for p_2, p_{1m}, π_1 , and π_2 . We describe our approach next.

First, we solved the problem by assigning k a couple of specific numerical values, i.e., $k = 1, 2, 3$. This can be done easily using a standard software package (we used Maple). However, cannot be replicated for a general k . Given the solutions for p_2 we obtained, we then guessed the general form of p_2 as a function of k . Finally, we verified that this is indeed the (unique) solution.

Here we present in detail the steps outlined in the above paragraph. We set $t = 1$ and $k = 1, 2, 3$, and we maximized (A6) with respect to p_2 . This yields $p_2 = .75$ (when $k = 1$), $p_2 = .625$ (when $k = 2$), and $p_2 = .5625$ (when $k = 3$), respectively. Our guess for the general form of firm 2's price was

$$p_2 = t \left(\frac{1}{2} + 2^{-(k+1)} \right). \tag{A7}$$

Given (A7), we also can calculate the general functions for m_1 and m_2 . These are

$$m_1 = 3 \times 2^{(k-2)} - 1 \quad \text{and} \quad m_2 = 3 \times 2^{(k-2)} + 2, \quad \text{for } k \geq 2. \tag{A8}$$

When $k = 1$, $m_1 = 0$, and $m_2 = 3$. We plug m_1 and m_2 , as given by (A8), into (A6), and we perform the summation. This yields

$$\pi_2(p_2, k) = \frac{p_2 (2t + 2^{(1-k)}t - 2p_2)}{4t}. \tag{A9}$$

Notice that in deriving (A9) we do not allow m_1 and m_2 to vary when p_2 varies. In other words, we assume that the structure does not change with p_2 . This might be true only for local changes of p_2 , as m_1 and m_2 are functions of p_2 , and a sizeable change in p_2 will change the value of m_1 and m_2 . We differentiate (A9) with respect to p_2 , and then we plug in (A7) to obtain

$$\left. \frac{\partial \pi_2(p_2, k)}{\partial p_2} \right|_{p_2=t(\frac{1}{2}+2^{-(k+1)})} = 0. \tag{A10}$$

Now note that (A10) would prove that (A7) is the unique profit-maximizing price, if the following two assumptions were true: (1) local changes of p_2 (around the guessed solution) do not change the m_1 and m_2 ; and (2) $\pi_2(p_2, k)$, as it is given by (A6), is continuous and strictly concave in p_2 . Then by the maximum theorem a unique maximum exists (p_2 lies in a compact set), and moreover it is characterized completely by (A10). Next, we prove that these two assumptions are indeed true.

- (3) Step 3. By plugging p_2 [as given by (A7)] into m^* and m^{**} [as they are given by (A2) and (A4), respectively], we obtain

$$m^* = \frac{(3 \times 2^k - 1)}{4} \quad \text{and} \quad m^{**} = \frac{(3 \times 2^k + 5)}{4},$$

which are clearly never integers. Therefore, a small change in p_2 will not change m_1 and m_2 . Finally, we prove that $\pi_2(p_2, k)$ is continuous and strictly concave in p_2 for any k .

We begin this proof by first noting (easy to see) that for any k , p_2 always should be within the closed interval $[0, V]$. The $[0, V]$ interval can be divided into a number of subintervals. Observe that p_2 can vary in two distinct ways, such that (1) m_1 and m_2 remain unchanged; and (2) either m_1 or m_2 , or both, change (jump) [see Figure (A1)]. First, we show that $\pi_2(p_2, k)$, as given by (A6), is strictly concave, assuming that we vary p_2 without changing $m_1(p_2, k)$ and $m_2(p_2, k)$. The left segments ($m \leq m_1$) are irrelevant (since firm 2's demand is zero), and the right segments ($m \geq m_2$) are linear in p_2 [see (A6) with

m_2 fixed]. Let us then look at the middle segments ($m_1 + 1 \leq m \leq m_2 - 1$), where

$$\pi_{2m} = \frac{p_2(2tm - 2^k p_2 - 2^k t + 2t)}{4t \times 2^k}$$

is the profit function in segment m , which is clearly strictly concave. This is true for any m in the middle segments, therefore, starting from any given m_1 and m_2 , and noting that the sum (over the middle m 's) of strictly concave functions is strictly concave, we have proved that $\pi_2(p_2, k)$ is strictly concave, provided that the structure does not change. Clearly it is also continuous. Of course, the structure changes at certain values of p_2 [as Figure (A1) clearly illustrates]. It might be true that at these p_2 's the profit function is not concave or is discontinuous. Therefore, it remains to show that $\pi_2(p_2, k)$ is concave and continuous even when changes of p_2 alter the structure.

The critical points are where the jumps in the two step functions occur. Fix k , and choose a p_2 (say \bar{p}) such that m^* and m^{**} are integers (i.e., exactly at a jump); that is, $m^* = m_1$ and $m^{**} = m_2$ (observe that m^* and m^{**} become integers simultaneously, as a function of p_2 for any fixed k). If p_2 increases slightly, then we move to $(m_1, m_2 + 1)$, and if p_2 decreases slightly, then we move to $(m_1 - 1, m_2)$ (consult Figure A1). At $p_2 = \bar{p}$ the profit function is

$$\pi_2(\bar{p}, k) = \left[\sum_{m=m_1+1}^{m_2-1} \frac{m}{2^k} - \frac{\bar{p} - p_{1mM}(\bar{p}, k) + t}{2t} + \sum_{m=m_2}^{2^k} \frac{1}{2^k} \right] \bar{p}.$$

Suppose p_2 increases by ε , i.e., $p_2 = \bar{p} + \varepsilon$. The profit function now can be written as

$$\begin{aligned} \pi_2(\bar{p} + \varepsilon, k) = & \left[\sum_{m=m_1+1}^{m_2} \frac{m}{2^k} - \frac{(\bar{p} + \varepsilon) - p_{1mM}((\bar{p} + \varepsilon), k) + t}{2t} \right. \\ & \left. + \sum_{m=m_2+1}^{2^k} \frac{1}{2^k} \right] (\bar{p} + \varepsilon). \end{aligned}$$

Observe that the limit of the summation has changed after the price increase. The right derivative of the profit function at $p_2 = \bar{p}$ is

$$\frac{\partial \pi_2(\bar{p})}{\partial p_2} = \lim_{\varepsilon \rightarrow 0^+} \frac{\pi_2(\bar{p} + \varepsilon, k) - \pi_2(\bar{p}, k)}{\varepsilon} = \lim_{\varepsilon \rightarrow 0^+} \frac{2^{-k}t + t - 2\bar{p} - \varepsilon}{2t}.$$

Analogously, the left derivative is

$$\frac{\partial \pi_2(\bar{p}-)}{\partial p_2} = \lim_{\varepsilon \rightarrow 0^-} \frac{\pi_2(\bar{p}, k) - \pi_2(\bar{p} - \varepsilon, k)}{\varepsilon} = \lim_{\varepsilon \rightarrow 0^-} \frac{2^{-k}t + t - 2\bar{p} + \varepsilon}{2t}.$$

The left and right first derivatives are the same at any p_2 such that m^* and m^{**} are integers, i.e., $\partial \pi_2(\bar{p}+)/\partial p = \partial \pi_2(\bar{p}-)/\partial p = (2^{-k}t + t - 2\bar{p})/(2t)$. Now notice two things about π_2 : (1) it is continuous at \bar{p} , since it is differentiable; and (2) it is strictly concave at \bar{p} , since the first derivative is a decreasing function of price at that point. These results, coupled with the fact that π_2 is strictly concave and continuous for fixed m_1 and m_2 , imply that firm 2's profit function is strictly concave and continuous in p_2 .

By performing the summation in (A6), after we plug in (A7) and using (A8), we obtain a closed-form expression for firm 2's profit function:

$$\pi_2(k) = \frac{t(1 + 2 \times 2^{-k} + 4^{-k})}{8}.$$

Firm 1's profit function from (A5), after we plug in (A7) and using (A8) again, is

$$\pi_1(k) = \frac{t(9 - 6 \times 2^{-k} + 5 \times 4^{-k})}{16}. \quad \square$$

REFERENCES

- Allenby, G. M. and P. Rossi, 1999, "Marketing Models of Consumer Heterogeneity," *Journal of Econometrics*, 89, 57–78.
- Anderson, S. and A. de Palma, 1988, "Spatial Price Discrimination with Heterogeneous Products," *Review of Economic Studies*, 55, 573–592.
- Bailey, J., 1998, "Internet Price Discrimination: Self-Regulation, Public Policy, and Global Electronic Commerce," Working paper, The Robert H. Smith School of Business, University of Maryland.
- Banks, J. and S. Moorthy, 1999, "A Model of Price Promotions with Consumer Search," *International Journal of Industrial Organization*, 17, 371–398.
- Besanko, D., J.-P. Dubé and S. Gupta, 2003, "Competitive Price Discrimination Strategies in a Vertical Channel with Aggregate Data," *Management Science*, 49, 1121–1138.
- Bester, H. and E. Petrakis, 1996, "Coupons and Oligopolistic Price Discrimination," *International Journal of Industrial Organization*, 14, 227–242.
- Bhaskar, V. and T. To, 2004, "Is Perfect Price Discrimination Really Efficient? An Analysis of Free Entry Equilibria," *RAND Journal of Economics*.
- Chen, Y., 1997, "Paying Customers to Switch," *Journal of Economics & Management Strategy*, 6, 877–897.
- Chen, Y., C. Narasimhan and Z.J. Zhang, 2001, "Individual Marketing with Imperfect Targetability," *Marketing Science*, 20, 23–41.
- Corts, K., 1998, "Third-Degree Price Discrimination in Oligopoly," *RAND Journal of Economics*, 29, 306–323.

- Fudenberg, D. and J. Tirole, 2000, "Customer Poaching and Brand Switching," *RAND Journal of Economics*, 31, 634–657.
- Kats, A., 1987, "Location-Price Equilibria in a Spatial Model of Discriminatory Pricing," *Economics Letters*, 25, 105–109.
- Lederer, P. and A.P. Hurter, Jr., 1986, "Competition of Firms: Discriminatory Pricing and Location," *Econometrica*, 54, 623–640.
- Matthews, S., 1984, "Information Acquisition in Discriminatory Auctions," in M. Boyer and R.E. Kihlstrom, eds., *Bayesian Models in Economic Theory*, Amsterdam, Holland: Elsevier Science Publishers, 181–207.
- Rao, R., 1991, "Pricing and Promotions in Asymmetric Duopolies," *Marketing Science*, 10, 131–144.
- Rossi, P., R.E. McCulloch and G. M. Allenby, 1996, "The Value of Purchase History Data in Target Marketing," *Marketing Science*, 15, 321–340.
- Shaffer, G. and Z.J. Zhang, 1995, "Competitive Coupon Targeting," *Marketing Science*, 14, 395–415.
- and —, 2000, "Pay to Switch or Pay to Stay: Preference-Based Price Discrimination in Markets with Switching Costs," *Journal of Economics & Management Strategy*, 9, 397–424.
- and —, 2002, "Competitive One-to-One Promotions," *Management Science*, 48, 1143–1160.
- Thisse, J. F. and X. Vives, 1988, "On the Strategic Choice of Spatial Price Policy," *American Economic Review*, 78, 122–137.