Predictive Analytics and Ship-then-shop Subscription

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Abstract

This paper studies an emerging subscription model called ship-then-shop. Leveraging its predictive analytics and artificial intelligence (AI) capability, the ship-then-shop firm curates and ships a product to the consumer, after which the consumer shops (i.e., evaluate product fit and make a purchase decision). The consumer first pays the upfront ship-then-shop subscription fee prior to observing product fit and then pays the product price afterwards, if she decides to purchase. We investigate how the firm balances the subscription fee and product price to maximize its profit when consumers can showroom. A key finding is the ship-then-shop firm's non-monotonic surplus extraction strategy with respect to its prediction capability. As prediction capability increases, the firm first switches from *ex-ante* to *ex-post* surplus extraction (by lowering fee and raising price). However, if the prediction capability increases further, the firm reverts to *ex-ante* surplus extraction (by raising fee and capping price). We also find that the ship-then-shop model is most profitable when *(i)* the prediction capability is advanced, *(ii)* the search friction in the market is large, or *(iii)* the product match potential is large. Finally, we show that the marginal return of AI capability on the firm's profit decreases in search friction but increases in product match potential. Taken together, we provide managerially relevant insights to help guide the implementation of the innovative subscription model.

Keywords: predictive analytics, artificial intelligence, subscription business, ship-then-shop, free-riding, showrooming, ex-ante and ex-post extraction

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1 Introduction

Advances in machine learning techniques and data digitization have catalyzed firms' interest in predictive analytics. Firms are fervently jumping on the prediction bandwagon (eMarketer 2021),[1] to optimize operations and marketing strategies (eMarketer 2020).[2] For example, financial service providers invest heavily in AI-powered chatbot services to improve customer relationships management[3], and tech firms deploy data-driven predictive analytics to recommend books to read (Amazon), jobs to apply for (LinkedIn), and friends to contact (Meta).[4] Enhancements in prediction capabilities not only improve the outcomes of firms' pre-existing marketing strategies, such as customer retention and product recommendation, but also motivate firms to qualitatively reinvent their business models. For instance, Agrawal et al. (2018) discuss the vast potential for predictive analytics to transform firms' business models; they predict the emergence of an innovative retail strategy called *ship-then-shop* subscription service. In this paper, we investigate this innovative business model that is increasingly gaining traction in practice.

Traditionally, the online shopping process starts with consumer search. The consumer searches for product information, browses various offerings, and evaluates product fit. If the consumer purchases, the firm ships the product and the shopping process terminates. In contrast, under the ship-then-shop model, the shopping process begins with product shipment. The firm leverages the prediction machine to predetermine products that match the consumer's taste and ships the product to her. The consumer then evaluates product fit, and decides whether to purchase or return the product (see Figure 1).

A unique feature of the ship-then-shop model is the separation of payments before and after the consumer learns product match. The consumer first pays the upfront service fee[5] prior to observing product fit, and then conditional on subscription, decides product purchase after observing product fit. Shopping assistants have played (and still do in many sectors) a similar role in improving product matches (Wernerfelt, 1994). Nevertheless, the recent rise of the new retail format, ship-then-shop, is largely propelled by advances in automated prediction technology. The technology allows firms

[1] `https://bit.ly/3H2Wpd8`

[2] eMarketer predicts "worldwide revenues for big data analytics – including predictive analytics and consumer scoring – to grow by nearly 450% to reach $68.09 billion in revenues by 2025" (`https://bit.ly/3H2Wpd8`).

[3] `https://go.td.com/3rYkTQr`

[4] *Financial Times* 2016 `https://on.ft.com/3JEw5I4` (accessed July 17, 2022)

[5] The service fee can be either a subscription fee or a styling fee. For example, Stitch Fix, an online styling service provider, charges $20 styling fee for assembling the curation box (as of March 2022). Even though subscription is not required, a consumer must pay the upfront fee to receive the product and learn its match value.

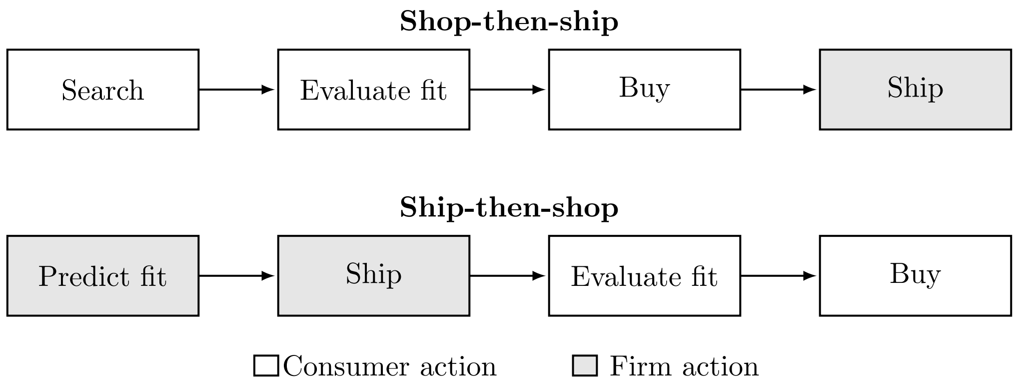


Figure 1: Two Modes of Online Shopping

to serve consumers with predictive delivery boxes at low costs. Without drastic improvements in prediction capability, the new retail format would not have been viable due to high product return rates from poor matches and inefficient cost structures where the firm incurs labor costs for serving every customer via human shopping assistants. In sum, a critical determinant of the ship-then-shop model's profitability is a sophisticated prediction machine.

Until recently, the idea of such predictive shipping has been dismissed by critics as hype (Banker, 2014; DePuy, 2014). However, the emergence of ship-then-shop subscription business models, notably in apparel retail sectors (Moore 2020; McKinsey & Company, 2018[6]), suggests Agrawal et al. (2018)'s prediction about the emergence of ship-then-shop subscription service is steadily unfolding in reality. The "best exemplar" of the ship-then-shop subscription provider is the apparel company Stitch Fix (Sinha et al., 2016). Stitch Fix leverages its prediction algorithm to predetermine consumers' style preferences and ships personalized clothing items. Consumers try on the clothes and decide whether to purchase or return the products. Trunk Club offers a similar "try before you buy" subscription model, whereby the company deploys sophisticated algorithms "to predict the most likely fit for a consumer" and then ships personalized "clothing subscription boxes."[7] Amazon is expanding its "Prime Try Before You Buy" service for its Prime subscribers which uses "a combination of technology innovation and a human touch to curate items." The product category for "Prime Try Before You Buy" ranges from apparel and shoes to accessories and jewelry.[8] Beyond apparel, tea subscription provider "Sips by" and trendy lifestyle subscription company "FabFitFun" customize packaging and ship new tea flavors or wellness and home items based on customer's preferences.[9]

[6] https://mck.co/3nujyxe
[7] https://bit.ly/3qc2bm4
[8] https://bit.ly/3FnBYaP
[9] https://bzfd.it/3SO2Hjn

Despite the increasing adoption of ship-then-shop models, little is understood about their economics. Standard two-part tariff solution dictates that firms should choose low product prices to reduce distortion and extract the surplus through high upfront fees; i.e., *high-fee-low-price* strategy (e.g., Clay et al., 1992; Essegaier et al., 2002). Extending this logic to the ship-then-shop model, one may intuit that as AI's prediction capability increases, the firm should raise its subscription fee and lower product price. However, this intuition does not always carry over due to fundamental differences between the ship-then-shop subscription and traditional subscription programs. First, the ship-then-shop subscription that we study focuses on curation subscription rather than replenishment subscription; the subscription value arises primarily from new product discovery. Second, unlike standard subscription settings where product value is fixed, in the ship-then-shop model, the firm's prediction machine helps improve the product match value, which is *ex-ante* unobservable to consumers but materially impacts the firm's strategies. Lastly, ship-then-shop subscription enables consumer service free-riding, whereby consumers identify the product fit through ship-then-shop service and then purchase the same product elsewhere at a lower price (Shin, 2007). There is less scope for free-riding or showrooming behavior in traditional subscription programs where the service and sales are inseparable (e.g., mobile service). Taken together, it is not clear how the firm should balance the subscription fee and product price to maximize its profit, especially in a setting where consumers may showroom.

We develop a parsimonious theoretical framework to elucidate the key economic forces that shape the firm's strategies under the ship-then-shop model. We demonstrate how the firm balances two revenue channels (ship-then-shop subscription and product sales), jointly optimizing subscription volume vs. upfront fee from all consumers, and product sales volume vs. product margin from the ship-then-shop subscribers. Moreover, we discuss how the firm's optimal strategies and profit vary with advances in prediction capability under different market conditions.

The central finding of the paper is that the firm's optimal strategy depends crucially on the trade-off between *ex-ante* vs. *ex-post* surplus extraction. Relative to traditional shopping, ship-then-shop provides two benefits to consumers: superior product match (matching effect) and search cost reduction (convenience effect). In making their subscription decisions, consumers weigh the benefits of the matching effect (which increases consumers' *ex-post* product valuation) and the convenience effect (which increases consumers' *ex-ante* valuation of ship-then-shop program) against the subscription fee and product price. Moreover, consumers are rational and consider showrooming; i.e., they potentially free-ride off of the ship-then-shop matching service to identify a high-match-value product and then purchase the same product elsewhere at a lower price. The resolution of this trade-off depends on the firm's prediction capability and the degree of search friction in the market.

As prediction capability increases, such that the expected match value of the shipped product increases, the firm initially lowers the service fee and raises the product price; i.e., it shifts from *ex-ante* to *ex-post* surplus extraction strategy. The intuition revolves around the interplay of the matching effect and convenience effect. If the AI's prediction capability is low, the matching effect is correspondingly low such that the firm sets a high subscription service fee to extract the consumers' *ex-ante* surplus generated by the convenience effect. This high-fee-low-price strategy is qualitatively similar to the standard two-part tariff solution. On the other hand, if the prediction capability increases, the matching effect dominates the convenience effect. The firm lowers the subscription fee to entice consumers to subscribe, and then through high product price extracts *ex-post* surplus generated by the matching effect. If the prediction capability becomes sufficiently advanced, the firm can charge an even higher price to extract additional surplus. However, high product price may prompt ship-then-shop subscribers to showroom, creating interesting dynamics between sales volume and product margin. The potential to showroom disciplines the firm and exerts downward pressure on product price, such that for high prediction capability, the firm reverts to increasing the fee.

We further characterize the conditions under which the ship-then-shop model is most profitable. We find that the firm's profit increases in *(i)* its prediction capability, *(ii)* the degree of search friction in the market, and *(iii)* the product match potential. Intuitively, consumers' valuations of the matching effect and convenience effect increase in the prediction capability and search friction. Also, greater product match potential increases the upside gain from the matching effect such that the firm's profit increases. We further show that the marginal return of AI's prediction capability on the firm's profit decreases in search friction, but increases in product match potential. The negative interaction between matching and convenience effects provides important managerial insights. For instance, if the firm operates in a market characterized by high search friction, its primary revenue source is the convenience effect, such that improving its prediction capability yields low marginal return. In such cases, the firm should turn more to improving the convenience effect rather than to improving the matching effect (e.g., investments in AI technology). On the other hand, if the product match potential is large, it is in the firm's best interest to invest in improving its prediction technology, which yields a higher marginal return than enhancing the convenience effect.

The rest of the paper is organized as follows. The next section discusses related literature. Section 3 describes the main model, and and Section 4 presents the analysis and main results. In

Section 5, we demonstrate the robustness of our main insights by analyzing several extensions that relax key model assumptions. Section 6 concludes with discussions of future research. For ease of exposition, we relegate all proofs and lengthy algebraic expressions to the appendix.

2 Related Literature

This paper lies at the intersection of several research streams: the effect of predictive analytics and economics of AI, recommendation system and targeting, and consumer search. The core driver of ship-then-shop service is the firm's ability make accurate, data-driven predictions about consumer preferences. Our model complements the literature on data-driven services such as recommender systems and content personalization. Many marketing papers focus on methodologies for improving recommender systems and data-driven personalization. Ansari et al. (2018) explore dynamic recommender systems under a hierarchical Bayesian framework. Lu et al. (2016) tackle recommender systems optimization using unstructured data extraction. Hauser et al. (2009) demonstrate the value of website personalization based on inferred consumer preferences and cognitive styles. Yoganarasimhan (2020) applies personalization to query-based search and demonstrates the value of machine learning algorithms in ranking search results, taking into account users' search and click history. In contrast, our paper focuses on the implications of firms' AI adoption on their strategies and profits. Enhancements in prediction capability motivate firms to transform their business models and adopt innovative business format, ship-then-shop, which is the main focus of our paper.

Our paper is also related to previous literature that studies the effects of targeting accuracy improvements on equilibrium outcomes.[10] Early work by Chen et al. (2001) shows that imperfect targeting can soften price competition among firms. Iyer et al. (2005) find that targeted advertising can improve firm profits compared to a no-targeting benchmark as it creates endogenous differentiation. Bergemann and Bonatti (2011) study targeting with a focus on the advertising market; they highlight the implications of targeting improvements on advertising price. Recent studies investigate the implications of targeting accuracy on different aspects of marketing. Shin and Yu (2021) examine how the mere fact that consumers are targeted by advertisements can affect consumer inference about product match and their subsequent search behaviors. From a platform design perspective, Zhong (2022) studies the effect of product-buyer match precision on consumer search, firms' prices, and

[10]A stream of research on the behavior-based pricing also examines the effect of targeted pricing using customers' past purchase history, which improves targeting accuracy (Fudenberg and Tirole, 2000; Shin and Sudhir, 2010; Villas-Boas, 1999, 2004).

platform revenue. Ichihashi (2020) shows that a firm's commitment to forego consumer information in its pricing decisions alleviates price-discrimination concerns, encouraging consumers to disclose information; this in turn helps the firm improve product targeting and ultimately increases its profit. Ning (2018) studies consumers' privacy choices under different pricing regimes. As prediction capability increases, targeted ads serve as an implicit product recommendation. Simultaneously, it exacerbates misaligned incentives to exploit this recommendation role to affect consumer's decision. Thus, more accurate targeting can raise prices and lower consumer welfare, which incentivizes consumers to opt out of data collection. Choi et al. (2022) consider more direct consumer privacy costs from compromised privacy. They show that consumers may opt in to online tracking to increase the efficiency with which advertisers show targeted ads along the consumers' purchase journey; this reduces wasteful ad repetition that causes consumer wearout. Similar to these papers, we study the effect of targeting accuracy on the firm's equilibrium strategy. However, our focus is on the implications of targeting accuracy on the profitability of an emerging retail format – namely, ship-then-shop business – and its optimal pricing strategy. We characterize the relationship between firms' prediction capability and the trade-off between matching effect and convenience effect, which ultimately shapes consumers' subscription choices.

Our model allows consumer showrooming.[11] After identifying a high-match product from the ship-then-shop firm, consumers may switch to buying the same product from the traditional market at a lower price.[12] Shin (2007) is the first paper that formally analyzes consumer showrooming and its interaction with retailer competition. It shows that consumer free-riding can soften price competition, making both retailers better off compared to the case without consumer free-riding. Other studies investigate the effects of showrooming in the context of competition between offline and online retailers (Jing, 2018; Mehra et al., 2018), and between manufacturers and retailers (Kuksov and Liao, 2018). Most papers highlight negative effects of free-riding on firm profits (Jing, 2018) and provide practical suggestions to counter showrooming – for example, matching online competitor's price, and offering exclusive product assortments (Mehra et al., 2018). Kuksov and Liao (2018) show that under endogenous manufacturer–retailer contract, consumer showrooming may increase profitability because the manufacturer may compensate the retailer for the informational services it

[11]While the showrooming literature focuses on consumers' free-riding behaviors, a large literature examines the firm's free-riding behaviors of the competing firms marketing efforts, such as advertising (Lewis and Nguyen, 2015; Lu and Shin, 2018; Shapiro, 2018).

[12]In most showrooming literature, the service-providing firm charges a higher price due to the cost disadvantage arising from selling costs associated with sales service (Shin, 2005). A similar force is at play in our model and induces the ship-then-shop firm to charge a higher price. However, the upward price pressure stems more from AI-based improvements in match value than from cost-side effects.

provides. Bar-Isaac and Shelegia (2020) shows that facilitating showrooming can have mixed effects on prices depending on the retailer format and the types of consumers who showroom. We contribute to this literature by investigating the effect of showrooming on the ship-then-shop service provider's pricing decision. We find that showrooming prevents the ship-then-shop firm from overcharging, especially when the firm's prediction capability is sufficiently advanced. This showrooming effect qualitatively alters how the firm optimally balances its upfront fee and product price.

Finally, the consideration of subscription fee and product price in our model resembles a two-part tariff. Two-part tariffs have been studied extensively as a tool for price-discrimination among consumers with heterogeneous usage rates. The literature explores variants of two-part tariffs where consumers can either select plans before or after their demand is realized (Clay et al., 1992), and compares two-part tariffs to alternative pricing schemes such as price-quantity bundles (Kolay and Shaffer, 2003). A common theme in this literature is that a high upfront fee and low prices are optimal (Oi, 1971). Firms extract surplus through high fixed fees and induce consumers to purchase larger quantities by reducing price distortion.[13] Our paper is different in that the firm's surplus extraction via subscription fee is *ex-ante* (prior to product match value realization), while extraction via product price is *ex-post* (post product match value realization). In addition, our model connects the literature on two-part tariff and showrooming, and derives novel insights at their intersection. For instance, we find that the firm adopts *low-fee-high-price* strategy if its prediction capability is intermediate, which is in stark contrast to the standard two-part tariff strategy. We further show that improvements in firm's prediction accuracy may increase consumers' incentive to free-ride such that the firm reverts to increasing the upfront fee.

3 Model

We consider a monopolist firm and a unit mass of consumers. The firm offers ship-then-shop subscription, whereby it curates a large selection of products from the market and, based on its AI algorithm, predicts and ships best-matching products to its subscribers.[14] Consumers purchase one unit of the product through one of two shopping methods. They can either purchase in the traditional market through their own search, or they can subscribe to ship-then-shop. Importantly, the

[13] Essegaier et al. (2002) show that a monopolist may offer a negative entry fee under limited capacity (also known as "sign up bonus") to discriminate among heavy and light users. Our model sheds light on a novel incentive for the ship-then-shop firm to offer a "sign up bonus"; namely, to compensate for potentially poor product match when the firm's prediction capability is low (see Section 5.1).

[14] We use the term AI broadly to refer to the AI's prediction capability. Thus, we use the terms "prediction capability" and "AI capability" interchangeably.

two shopping methods result in different product match qualities, on which we elaborate below.

Firm

The firm offers ship-then-shop subscription service and makes two decisions: it sets the subscription fee F and product price p_s, where subscript s denotes *subscription*. We assume that the firm procures products from the market at price p_m, where subscript m denotes *market*. The firm then resells the best-matching products to ship-then-shop subscribers at price p_s, where $p_s - p_m$ is the firm's profit margin (i.e., profit premium if $p_s - p_m$ is positive, or profit loss if it is negative) it can charge for its ship-then-shop service. The market price p_m effectively serves as the wholesale price.[15] The firm's profit consists of two revenue sources, ship-then-shop subscription and product sales:

$$\mathbb{E}[\pi] = N_s \left(F + D_p \cdot (p_s - p_m)\right), \tag{1}$$

where N_s denotes the number of ship-then-shop service subscribers, F the subscription service fee, and D_p the demand for the shipped product.

Consumers

Consumers make two sequential decisions: subscription and product purchase. After observing the service fee F and product price p_s, consumers decide whether to subscribe to ship-then-shop or search in the traditional market.[16]

If consumers search in the traditional market, they incur search cost $s \in \{s_L, s_H\}$ to discover the product and realize its match value

$$v_m \sim U[0, V], \tag{2}$$

where V denotes the maximum attainable match value−it can also be interpreted as the product match potential in a given market. Consumers then decide whether to purchase the product at price $p_m \in [0, V]$.[17]

[15] We allow p_s to be different from p_m; for example, tea subscription provider "Sips by" (https://www.sipsby.com/) and knitting subscription provider "KnitCrate" (https://www.knitcrate.com/) customize packaging and shipping such that product prices are likely to be different from comparable products in the traditional market. It is also worth noting that we do not restrict the product price p_s be higher than p_m.

[16] In Section OA1.3 of the Online Appendix, we analyze a scenario in which p_s is unobservable to consumers prior to product receipt. We show that the qualitative insights carry over.

[17] Consumers can return products in the traditional market free of charge; in practice, many retailers implement

On the other hand, if consumers subscribe to ship-then-shop and receive the shipped product, consumers realize product match value

$$v_s \sim U[\alpha V, V], \tag{3}$$

where $\alpha \in [0, 1]$ captures the firm's prediction capability or matching quality. To illustrate the role of α, if $\alpha = 0$, then the firm's match prediction capability is no better than the consumer's own ability to identify product matches through her search. On the other hand, if $\alpha = 1$, then the firm perfectly identifies and ships the consumer's ideal product, in which case the consumer obtains the maximum match value V. Thus, AI capability shifts the consumers' product match value distribution upwards − we call this the *matching effect*.

Here, we assume that the firm's prediction capability is superior to the consumer's capability in expectation (i.e., $\alpha \geq 0$). There are supply-side and demand-side arguments that support this assumption. On the supply side, firms may leverage sophisticated prediction algorithms by pooling data from other customers in a collaborative manner. Such prediction algorithms can generate customer insight, of which consumers themselves may not be aware; firms can predict consumers' needs and wants before consumers recognize them. On the demand side, consumers may have limited knowledge about the supply-side. Even if consumers know their preferences, they may not know what products are available in the market, still less whether these products fit their needs and wants. Nevertheless, there may be cases where the firm's prediction capability is inferior to the consumers' capability in expectation. For example, experts among consumers may possess human intuition and judgment that machines fail to capture. In Section 5.1, we relax this assumption and allow the firm's prediction capability to be inferior to the consumers' (i.e., $\alpha \in [-1, 1]$); we demonstrate that the qualitative insights continue to hold.

Upon receiving the ship-then-shop product and realizing match value v_s, consumers choose between three actions: *(i)* purchase the product at price p_s, *(ii)* return the product at hassle cost h, which is not too large,[18] or *(iii)* return the product at hassle cost h and purchase the same product from the traditional market at price p_m. The third action is a form of free-riding or showrooming, in which the ship-then-shop subscriber receives the ship-then-shop firm's product matching service but purchases from a competing channel (Jing, 2018; Mehra et al., 2018; Shin, 2007). As we will discuss later, the potential for consumers to free-ride on the matching service exerts downward pressure on

consumer-friendly return policies that make return processes convenient for consumers. However, given that consumers resolve their product match uncertainty through search, they will not return purchased items in equilibrium.

[18] We assume that $h < p_m$.

the firm's product price p_s.

While consumers observe F and p_s, the product match values v_s and v_m are *a priori* unknown. Consumers observe v_s under ship-then-shop subscription only upon receiving the product, and v_m under the traditional shopping only after product search.

Consumer utility consists of two components: product consumption utility and product match value. Product consumption utility is common across all products in the same category, whereas product match value depends on the specific product that consumers find through either their own search or ship-then-shop recommendation. Thus, the consumer's utility is

$$u = u_0 + v, \tag{4}$$

where u_0 is the product consumption utility, and v the product match value. We normalize u_0 to zero without loss of generality. The consumer's product match value v depends on her choice of shopping method. If she subscribes to ship-then-shop after paying fee F, her product match value v_s is drawn from $U[\alpha V, V]$. If she searches in the traditional market, her product match value v_m is drawn from $U[0, V]$.

The consumer's net utility from subscribing to ship-then-shop is

$$u_s = -F + \delta \cdot \begin{cases} v_s - p_s & \text{if purchase from ship-then-shop firm,} \\ -h & \text{if return without purchase,} \\ -h + v_s - p_m & \text{if return and purchase from traditional channel,} \end{cases} \tag{5}$$

where $\delta \in (0, 1)$ is the discount factor, capturing the delayed product consumption under ship-then-shop. For ease of exposition, we hereafter set $\delta \to 1$.

On the other hand, the consumer's net utility from the traditional market is

$$u_m = -s + \begin{cases} v_m - p_m & \text{if purchase,} \\ 0 & \text{if not purchase,} \end{cases} \tag{6}$$

where $s \in \{s_L, s_H\}$ is the heterogeneous search cost.[19] Consumers are low-type ($s = s_L$) or high-type ($s = s_H$) with equal probability. We normalize s_L to zero without loss of generality; i.e., $0 = s_L <$

[19]We treat search costs primarily as costs associated with product discovery. Thus, we assume that when consumers return the ship-then-shop product and purchase from traditional channel, they do not incur search costs.

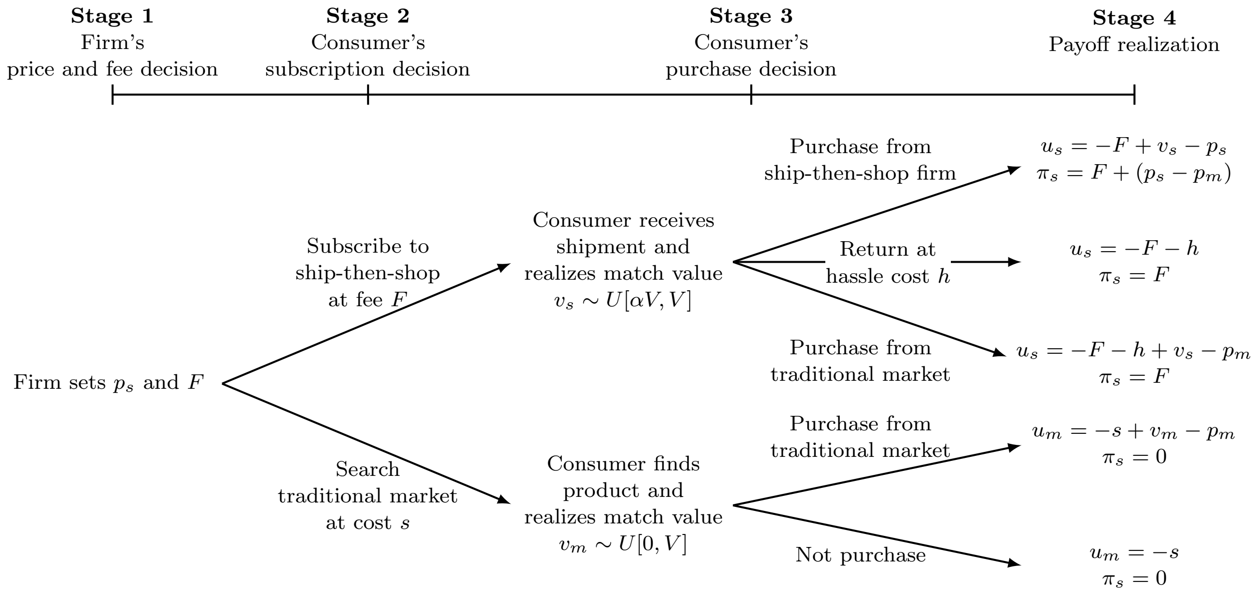


Figure 2: Game Sequence

s_H. Moreover, to focus on the more interesting case where all consumers consider searching in the traditional market, we assume that $s_H \leq (V - p_m)^2 / 2V$.[20] Note that if the consumer purchases from the ship-then-shop firm, she does not incur the search cost s (see (5)), whereas if she buys from the traditional market, she does (see (6)). That is, ship-then-shop subscription facilitates shopping by saving consumers' search cost.[21] We call this the *convenience effect*.

Overall, in making their subscription decisions, consumers weigh the potential gains from the matching effect (which increases consumers' *ex-post* product valuation) and the convenience effect (which increases consumers' *ex-ante* valuation of ship-then-shop program) against the cost of subscription and product price. The game sequence is summarized in Figure 2.

4 Analysis

We solve for subgame perfect Nash equilibrium using backward induction. We start from the last stage of consumer product purchase decision.

[20]If $s_H > (V - p_m)^2 / 2V$, the game degenerates to a trivial case where the high-type consumers do not consider buying in the traditional market.

[21]According to online reviews of Stitch Fix and Wantable, consumers who "hate shopping, but love new clothes" find "the process ... very convenient" as "it saves time." "[They] are paying for the convenience of not going out to shop" (`https://bit.ly/3Dr4wSI`).

4.1 Consumer Decision

Consumers' decisions are two-fold: ship-then-shop subscription and product purchase. Conditional on subscribing to ship-then-shop, consumers either *(i)* purchase from the ship-then-shop firm, which yields utility $v_s - p_s$, *(ii)* return the product at hassle cost h, which yields utility $-h$ or *(iii)* return the product at hassle cost h and then purchase the same product from the traditional market at price p_m, which yields utility $-h + v_s - p_m$.

As we later demonstrate, in equilibrium, the firm will always set product price

$$p_s \leq p_m + h, \tag{7}$$

such that consumers do not have incentive to free-ride. Consumers purchase the shipped product if and only if the realized match value net of price exceeds the disutility they incur from returning the product: $v_s - p_s \geq -h$. Note that if $p_s - h < \alpha V$, ship-then-shop subscribers always buy. Therefore, the marginal consumer who purchases is

$$\bar{v} \equiv \max \{\alpha V, p_s - h\}. \tag{8}$$

Consumers also decide whether to subscribe to ship-then-shop or search in the traditional market. Consumers' expected utility from subscribing to ship-then-shop is

$$\mathbb{E}[u_s] \quad = \quad -F + \left(\int_{\alpha V}^{\bar{v}} \frac{-h}{V(1-\alpha)} dv_s + \int_{\bar{v}}^{V} \frac{v_s - p_s}{V(1-\alpha)} dv_s \right) \tag{9}$$

The first term denotes the subscription service fee, and the second term in brackets the expected utility from either returning or purchasing the shipped product.

If consumers search in the traditional market at cost $s \in \{0, s_H\}$, their expected utility is

$$\mathbb{E}[u_m] = -s + \int_{p_m}^{V} \frac{v_m - p_m}{V} dv_m = -s + \frac{(V - p_m)^2}{2V}. \tag{10}$$

Consumers compare their expected utility from ship-then-shop subscription in (9) and that from traditional market in (10). Let $\bar{s}$ denote the search cost for which consumers are indifferent between the two shopping options. Solving $\mathbb{E}[u_s] = \mathbb{E}[u_m]$ yields

$$\bar{s} = F - \left(\frac{(V(2\alpha h + V) - \bar{v}(2h + \bar{v}) - 2p_s(V - \bar{v}))}{2V(1-\alpha)} - \frac{(V - p_m)^2}{2V} \right). \tag{11}$$

While consumers with search cost $s > \bar{s}$ subscribe to ship-then-shop, those with search cost $s \leq \bar{s}$ search in the traditional market. Specifically, *(i)* if $\bar{s} \leq 0$, then all consumers subscribe; *(ii)* if $0 < \bar{s} \leq s_H$, then high-type consumers ($s = s_H$) subscribe, while low-type consumers ($s = 0$) choose the traditional market; and *(iii)* if $s_H < \bar{s}$, then all consumers choose the traditional market.

4.2 Firm Decision

The firm sets product price and service fee in anticipation of consumers' subscription and purchase decisions. The firm's expected profit is

$$\mathbb{E}[\pi(p_s, F)] = N_s(p_s, F)\Big(\underbrace{F}_{\substack{\textit{ex-ante} \\ \text{surplus}}} + \underbrace{\frac{V - \bar{v}}{V(1-\alpha)}(p_s - p_m)}_{\substack{\textit{ex-post} \\ \text{surplus}}} \Big), \tag{12}$$

where $N_s(p_s, F)$ denotes the number of ship-then-shop subscribers, F the service fee, and the last term the expected margin from product sales. $\frac{V-\bar{v}}{V(1-\alpha)}$ is the probability that a ship-then-shop subscriber purchases the product. The firm procures the product at market price p_m and resells it at price p_s.

The firm's profit in (12) reveals two channels through which the firm extracts consumer surplus: *ex-ante* surplus extraction through service fee F (i.e., expected consumer surplus prior to product match value realization), and *ex-post* surplus extraction through product price p_s (i.e., consumer surplus post product match realization). Throughout the paper, we use the following terminology: whenever the firm, in response to some change in market characteristic, places more weight on the fee F (vs. the product price p_s) for surplus extraction, we say that the firm adopts *ex-ante* surplus extraction strategy. Conversely, if it places more weight on the price p_s, we say it adopts *ex-post* surplus extraction strategy.

The firm determines $N_s(p_s, F)$ by adjusting p_s and F. Given the binary search cost space (i.e., $s \in \{0, s_H\}$), the firm considers two demand regimes: partial coverage and full coverage. Under partial coverage, the firm induces only the high-type consumers ($s = s_H$) to subscribe to ship-then-shop: $N_s(p_s, F) = 1/2$. Under full coverage, it induces both the high- and low-type consumers ($s = 0$) to subscribe: $N_s(p_s, F) = 1$.

Under partial coverage ($N_s(p_s, F) = 1/2$), the firm's problem is

$$\begin{aligned} \max_{p_s, F} \quad & \mathbb{E}[\pi_{\text{part}}] = \frac{1}{2}\left(F + \frac{V - \bar{v}}{V(1-\alpha)}(p_s - p_m)\right) \\ \text{subject to} \quad & 0 < \bar{s}(p_s, F) \leq s_H \text{ and } p_s \leq p_m + h \end{aligned} \tag{13}$$

where $\bar{v}$ is the marginal consumer who purchases the product as defined in (8) and $\bar{s}$ is defined in (11). The (IC) constraint $0 < \bar{s}(p_s, F) \leq s_H$ ensures only the high-type consumers subscribe to ship-then-shop. The condition $p_s \leq p_m + h$ is not an exogenous assumption we imposed in the model. It follows from the fact that setting $p_s > p_m + h$ is dominated by $p_s \leq p_m + h$.[22] Intuitively, if the price is set too high, then the subscribers switch to the traditional market after receiving the ship-then-shop service, such that the firm's product sales is 0. Therefore, under partial coverage, the firm's optimal product price satisfies $p_s \leq p_m + h$. Solving (13) yields $F^*_{\text{part}}(p_s) = s_H + \frac{(V(2\alpha h + V) - \bar{v}(2h+\bar{v}) - 2p_s(V-\bar{v}))}{2V(1-\alpha)} - \frac{(V-p_m)^2}{2V}$, and $p^*_s = \min\left\{\max\left\{\alpha V + h, p_m\right\}, p_m + h\right\}$.[23]

Under full coverage ($N_s(p_s, F) = 1$), the firm's problem is

$$\begin{aligned} \max_{p_s, F} \quad & \mathbb{E}[\pi_{\text{full}}] = F + \frac{V - \bar{v}}{V(1-\alpha)}(p_s - p_m) \\ \text{subject to} \quad & \bar{s}(p_s, F) \leq 0 \text{ and } p_s \leq p_m + h, \end{aligned} \tag{14}$$

where the (IC) constraint $\bar{s}(p_s, F) \leq 0$ ensures both consumer types subscribe. Similar to partial coverage, the constraint $p_s \leq p_m + h$ prevents subscribers from showrooming. Following the reasoning above, we obtain that the optimal price under full coverage coincides with that under partial coverage, while the optimal fee under full coverage is $F^*_{\text{full}}(p_s) = F^*_{\text{part}}(p_s) - s_H$, such that the low-type consumers' (IC) constraint binds. The following lemma summarizes the optimal product price and subscription fee under each regime.

Lemma 1. *The firm's optimal product price under both partial and full coverage is* $p^*_s = \min\left\{\max\left\{\alpha V + h, p_m\right\}, p_m + h\right\}$.

Before solving for the optimal coverage choice, we highlight an important relationship between product price and service fee. Observe that under either coverage, the firm's best-response fee $F^*(p_s)$ is decreasing in p_s:

$$\frac{\partial F^*(p_s)}{\partial p_s} = -\min\left\{1, \frac{V + h - p_s}{V(1-\alpha)}\right\} \leq 0.\text{[24]} \tag{15}$$

This implies that the service fee and product price are strategic substitutes. In optimizing the product price, the firm not only trades off the usual margin vs. sales, but also considers whether to extract *ex-ante* surplus through F or to extract *ex-post* surplus through p_s. The latter trade-off

[22] See Claim 1 in the Online Appendix for the detailed proof.

[23] Again, note that we do not impose any constraints on p_s when solving for the firm's optimal price. The expression for p^*_s implies that the firm does not undercut the market price p_m in equilibrium. This outcome is consistent with industry practice where brands sometimes contractually restrict the ship-then-shop firm from undercutting in price, which could hurt brand equity. We thank an anonymous reviewer for raising this point.

[24] Note that the firm never sets $p_s > V + h$, for if $p_s > V + h$, none of the consumers will purchase.

constitutes a key force in the model. As we later demonstrate, whether the firm adopts *ex-ante* vs. *ex-post* surplus extraction depends crucially on the firm's AI capability (α) and search friction (s_H).

Lemma 2. *The firm's product price and service fee are strategic substitutes:* $\frac{\partial F^*(p_s)}{\partial p_s} \leq 0$.

Next, we determine the firm's optimal coverage, and thereby characterize the optimal service fee. The firm compares the optimal profits under partial and full coverage, which are, respectively,

$$\mathbb{E}[\pi^*_{\text{part}}] = \frac{1}{4} \cdot \begin{cases} \alpha V + 2s_H - \frac{p_m^2}{V} & \text{if } p_m \leq \alpha V + h, \\ \frac{\alpha\left(V^2 - 2V(p_m + s_H - h) + p_m^2\right) + h(h - 2p_m) + 2s_H V}{V(1-\alpha)} & \text{if } p_m > \alpha V + h, \end{cases} \tag{16}$$

and

$$\mathbb{E}[\pi^*_{\text{full}}] = \frac{1}{2V} \cdot \begin{cases} \alpha V^2 - p_m^2 & \text{if } p_m \leq \alpha V + h, \\ \frac{h^2 - 2h(p_m - \alpha V) + \alpha(p_m - V)^2}{1-\alpha} & \text{if } p_m > \alpha V + h. \end{cases} \tag{17}$$

Lemma 3. *The firm chooses full coverage if $\alpha > \tilde{\alpha}$ (or equivalently, $s_H \leq \tilde{s}$); otherwise, it chooses partial coverage.*[25]

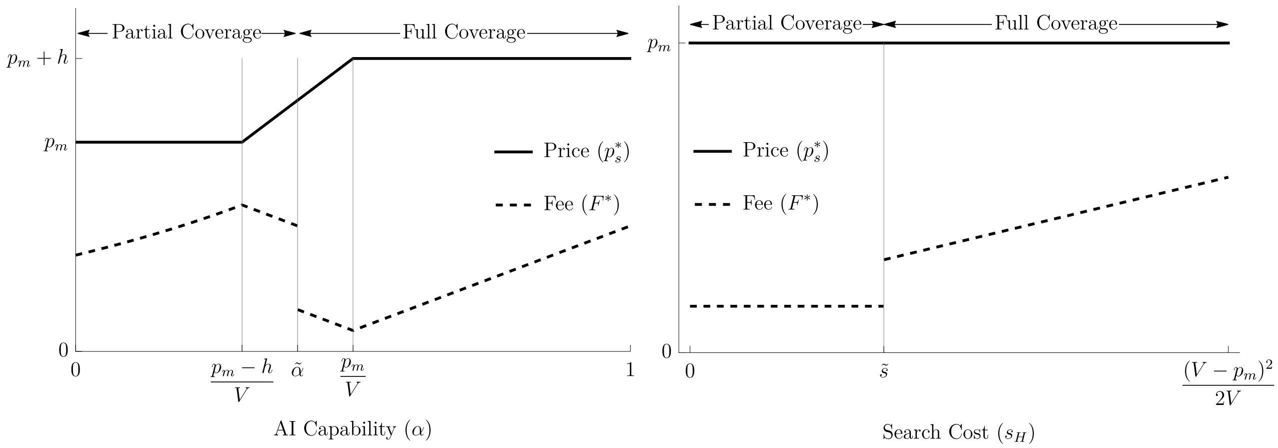
Lemma 3 shows that the firm chooses full coverage if its AI capability is sufficiently sophisticated or the search friction in the market is mild. Intuitively, larger α increases the ship-then-shop service's matching effect. Thus, even the low-type consumers, who can search in the traditional market at low search cost, have high valuation for ship-then-shop. Since the firm can charge a higher markup, the marginal benefit of increasing market coverage increases, such that the firm covers the whole market.

The firm also covers the whole market if search friction is mild (i.e., small s_H). Recall that the second benefit of ship-then-shop is the convenience effect: ship-then-shop facilitates consumer shopping by reducing their search costs. Therefore, if search friction is severe, the convenience effect increases such that the firm can extract large consumer surplus. In this case, the firm charges a high fee to extract the high-type consumers' surplus at the expense of forgoing subscription from low-type consumers.

Based on the firm's optimal coverage, we obtain the optimal price and fee:

$$p_s^* = \min\left\{\max\left\{\alpha V + h, p_m\right\}, p_m + h\right\}, \text{ and } F^* = F^*_{\text{part}}\left(p_s^*\right) - \begin{cases} 0 & \text{if } \alpha \leq \tilde{\alpha}, \\ s_H & \text{if } \alpha > \tilde{\alpha}. \end{cases} \tag{18}$$

[25]The thresholds $\tilde{\alpha}$ and $\tilde{s}$ are characterized in the proof.



(a) Impact of AI Capability ($s_H = 0.1$)

(b) Impact of Search Cost ($\alpha = 0.25$)

Figure 3: Impact of AI Capability and Search Cost on Price and Fee ($V = 1, p_m = 0.5, h = 0.1$)

The following proposition presents the main result concerning the interaction of the firm's equilibrium strategy (p_s^*, F^*) and *(i)* the firm's prediction capability, *(ii)* the degree of search friction in the market, and *(iii)* consumers' hassle costs.

Proposition 1. *The firm's equilibrium strategy* (p_s^*, F^*) *varies as follows.*

(i) With respect to α*: if* $\frac{p_m - h}{V} < \alpha \leq \frac{p_m}{V}$, p_s^* *increases in* α*; otherwise, it is constant in* α*.* F^* *varies non-monotonically in* α *with a discontinuous drop at* $\tilde{\alpha}$*.*[26] *Specifically,*

– *when* α *is either sufficiently low or high* ($\alpha \leq \frac{p_m - h}{V}$ *or* $\frac{p_m}{V} < \alpha$), F^* *increases in* α*;*

– *when* α *is in the intermediate range* ($\frac{p_m - h}{V} < \alpha \leq \frac{p_m}{V}$), F^* *decreases in* α*.*

(ii) With respect to s_H*:* p_s^* *is constant in* s_H*, while* F^* *weakly increases in* s_H*.*

(iii) With respect to h*:* p_s^* *weakly increases in* h*, while* F^* *decreases in* h*.*

If the firm's prediction accuracy is low (i.e., $\alpha \leq (p_m - h)/V$), the expected match quality is poor, which dampens consumers' valuation for ship-then-shop. In this range, the primary source of customer benefits is the convenience effect. Therefore, if α is small, the firm adopts *ex-ante* surplus extraction strategy. For intermediate level of prediction accuracy, the product match quality is sufficiently high that the primary source of customer benefit switches from the convenience effect to the matching effect. Thus, the firm changes its pricing strategy from *ex-ante* to *ex-post* surplus extraction; i.e., it raises the product price and lowers the subscription fee (see price and fee patterns as α increases in the interval $[0, p_m/V]$ in Figure 3a).

However, as the prediction accuracy increases further, the firm reverts to *ex-ante* surplus ex-

[26] $\tilde{\alpha}$ is the threshold value of α at which the firm switches from partial to full coverage (see Lemma 3).

traction strategy, even though the matching effect outweighs the convenience effect. In particular, for all $\alpha \in [p_m/V, 1]$, p_s^* remains constant while F^* increases in α. The rationale behind this reversion is that consumers' showrooming incentives increase as the product price increases. Consumers receive high-quality matching services from the ship-then-shop firm but purchase from the traditional market. To prevent sales loss from such free-riding, the firm caps the product price and raises the fee instead. In sum, the firm's extraction strategy follows a non-monotonic pattern with respect to the prediction accuracy, which contrasts sharply with the standard two-part tariff prescription of low-fee-high price. The firm adopts *ex-post* extraction for intermediate ranges of α, and *ex-ante* extraction for extreme ranges of α.

Also, the fee pattern with respect to s_H reflects the convenience effect, the second benefit of ship-then-shop subscription. Intuitively, the value of shopping without searching increases as searching in the traditional market becomes more costly. This increases consumers' valuation of ship-then-shop, which allows the firm to charge higher fee (see Figure 3b).

Finally, we find that $\frac{\partial p_s^*}{\partial h} \geq 0$ and $\frac{\partial F^*}{\partial h} < 0$. Higher hassle cost of returning unwanted products has a lock-in effect, which allows the firm to raise the product price. Thus, the firm extracts greater *ex-post* surplus by charging a high price, while it lowers its upfront fee to compensate for the risk of a product mismatch.

The implications of the matching and convenience effects for the firm's fee-price strategy described in Proposition 1 are robust to considerations of richer consumers' and the firm's decision-making. For example, the results of Proposition 1 carry over (a) to settings where the firm also incurs costs to process returned products, which may affect the firm's decision-making (see Section 5.2), and (b) under more flexible pricing schemes (see Section 5.3). In Section OA1.2 of the Online Appendix, we show that the firm's learning dynamics exert upward pressure on price while preserving the key insights from the main model. When the firm can learn from returned products and re-ship better matching products subsequently, the firm charges a higher price. The reason is that even if low-match consumers return the product, the firm can extract their surplus subsequently through learning.

4.3 Profitability of Ship-then-shop

We examine the profitability of ship-then-shop by conducting comparative statics of the firm's equilibrium profit with respect to firm-specific factor (e.g., prediction capability) and market-specific factors (e.g., severity of search friction and magnitude of product match potential).

Lemma 4. *The firm's expected profit (i) increases in α, s_H, and V; i.e., $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} > 0$, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} \geq 0$, and $\frac{\partial \mathbb{E}[\pi^*]}{\partial V} > 0$; and (ii) decreases in h; i.e., $\frac{\partial \mathbb{E}[\pi^*]}{\partial h} \leq 0$.*

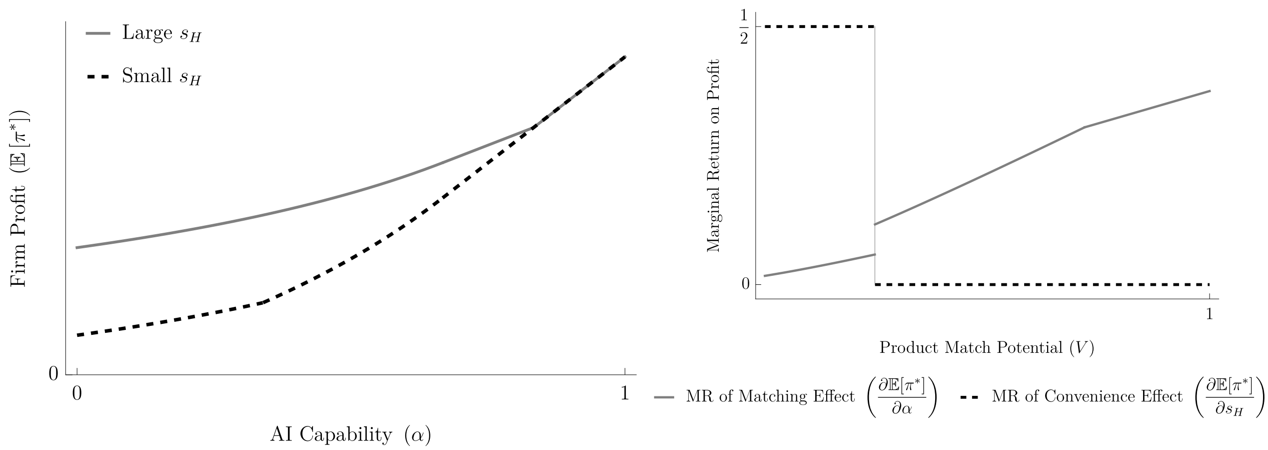
That the firm's profit increases in α and s_H is consistent with intuition. Equipped with higher AI capability, the firm provides a higher-quality match to consumers. Also, higher search friction in the traditional market increases the consumers' comparative valuations for ship-then-shop. Both enlarge the total surplus, which the firm extracts through the optimal fee-price combination in (18). Finally, as the maximum attainable match value V increases, the upside potential of the matching effect under ship-then-shop rises, increasing the firm's profit.

Given that the profitability of ship-then-shop increases in AI capability and market search friction, lay intuition may suggest that the firm should invest in enhancing both the matching and convenience effect. However, due to the linkage between *ex-ante* and *ex-post* consumer surplus, we find that the two effects are substitutes. In other words, returns from one effect diminishes the returns from the other.

Proposition 2. *The interaction effect of AI capability and search friction on the firm's profit is negative: $\frac{\partial}{\partial s_H}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}\right) \leq 0$. Moreover, the marginal return of α (s_H) on the firm's expected profit increases (decreases) in V; $\frac{\partial}{\partial V}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}\right) \geq 0$ and $\frac{\partial}{\partial V}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}\right) \leq 0$.*

While the firm's profit increases in α and s_H, the interaction effect of AI capability and search friction on the firm's profit is negative: $\frac{\partial}{\partial s_H}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}\right) \leq 0$. This suggests that for the firm offering ship-then-shop service, matching effect and convenience effect are substitutes. For instance, if the search friction in the market is severe such that the convenience effect is large, the marginal effect of AI capability on firm's profit diminishes. Intuitively, the firm capitalizes on the convenience effect by charging high service fees, thereby extracting the high-type consumers' surplus at the expense of foregoing demand from low-type consumers. As fewer consumers subscribe to ship-then-shop, the total returns from the matching effect decrease. We illustrate this effect in Figure 4a, which compares two scenarios: a large s_H and a small s_H. As α increases, the expected profit increases in both cases. However, the slope is steeper for small s_H. Under small search cost (represented by the dotted line in the figure), the firm switches to full coverage at low level of α. Thus, the firm benefits more from improvements in AI capability compared to the case of high search cost, wherein the firm switches to full coverage only at high level of .

Finally, Proposition 2 reveals an important insight regarding product match potential. The marginal return from the matching effect increases in product match potential, whereas the marginal



(a) Search Cost and Marginal Return

(b) Marginal Returns of Matching and Convenience Effects

Figure 4: Profitability of Ship-then-Shop

return from the convenience effect decreases in product match potential (see Figure 4b). If V is small, consumers face little product match uncertainty such that the matching effect under ship-then-shop adds little value. In this case, the ship-then-shop's value derives primarily from the convenience effect; therefore, the return from convenience effect dominates that from the matching effect. On the other hand, if V is large, the relationship reverses. The greater the product match potential, the greater the return from prediction capability because the firm can harvest the upside gains from the matching effect more effectively. In this case, the primary source of ship-then-shop's value is the matching effect. Therefore, the marginal return from the matching effect dominates that from the convenience effect.

The insights from Lemma 4 and Proposition 2 help explain the emergence of ship-then-shop business models in certain markets. Our analysis suggests ship-then-shop models are likely to be profitable if the firm's AI capability is advanced, the search friction in the target market is severe, or the consumers' product match potential is large. By and large, these findings are consistent with real-world observations that the emergence of ship-then-shop businesses has been concentrated in markets characterized by high search friction and product match potential; apparel categories (e.g., Stitch Fix, Trunk Club, and Prime Try Before You Buy) or accessories like glasses and shoes (Warby Parker, SneakerTub), whose fashion trends evolve quickly such that consumers entail high search costs or high match uncertainty. These firms ship products either with minimal consumer input during the curation process, lowering consumers' time and effort required for preference estimation (i.e., enhancing the convenience effect) or even with substantial consumer input in the curation process (e.g., uploading clothing images, completing extensive preference questionnaires, and communicating with “personal shoppers”), providing matching benefits based on their predictive analytics.

Our analysis can also help inform managerial decision-making under the ship-then-shop model. A ship-then-shop firm that considers investing in AI capability should be mindful of the source of marginal return on investment. For instance, it should exercise caution before rushing to improve its AI capability (e.g., hiring data scientists). In markets characterized by high search friction, the firm should consider focusing on improving the convenience of the consumers’ purchase process (e.g., by reducing the time and effort required for gathering consumer information for preference estimation). On the other hand, in product categories with large product match potential, it is in the firm’s best interest to invest in AI capability improvements, which yield higher marginal returns than enhancing the convenience effect.

5 Extensions

In this section, we relax several assumptions imposed in our main model to show the generalizability and robustness of our main results. Specifically, we consider *(i)* the possibility that the ship-then-shop firm’s prediction capability is, in expectation, inferior to the consumer’s ability to discover products through her own search, *(ii)* a scenario in which the firm incurs a cost to process return products, *(iii)* an alternative pricing scheme in which the ship-then-shop firm allows the upfront fee the consumers had paid upon subscription to be credited towards product purchase, and *(iv)* other extensions. Overall, we find that the qualitative insights from the main model carry over to these extension models.

5.1 Inferior Prediction Capability

In the main model, we assumed $\alpha \geq 0$ such that the firm’s prediction machine outperformed *in expectation* the consumer’s own search abilities. However, prediction requires substantial training data to attain a reasonable level of accuracy. For example, prediction accuracy would be inferior for firms that recently entered the market and thus lack data. To capture this possibility, we relax the positivity assumption of α and examine the extent to which our core insights carry over for negative α. We allow α to range from -1 to 1 instead of from 0 to 1.

We find that the optimal product price and subscription fee from the main model generalize to $\alpha \in [-1, 1]$. Technically, this is because the analysis does not depend on the positivity assumption of α. For instance, consistent with Lemma 3, the firm adopts partial coverage for negative α to leverage

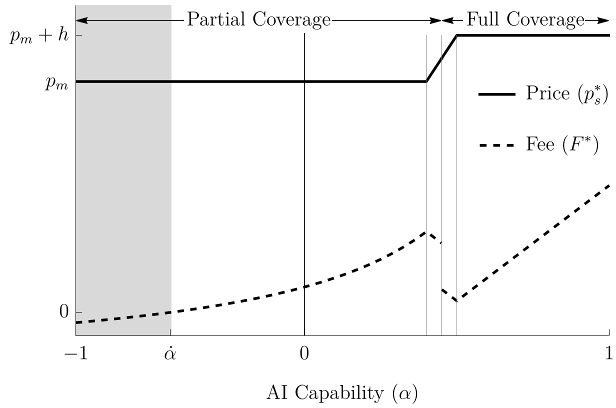


Figure 5: Impact of AI Capability on Price and Fee For $-1 \le \alpha \le 1$ ($V = 1, p_m = 0.5, h = s_H = 0.1$)

the convenience effect (vs. the matching effect) through high subscription fees.

An additional insight we obtain from the analysis of negative α is that the firm may charge negative subscription fee for small α (see Figure 5). This occurs if consumers' hassle cost (h) of returning unwanted products, is large. Intuitively, if α is small and h large, consumers discount the value of ship-then-shop: match will likely be poor, and the hassle of returning products large. Therefore, the firm offers compensatory "sign-up bonus" in the form of negative subscription fees. We state this result in the following proposition.

Proposition 3. *If h is sufficiently large, then there exists a threshold $\dot{\alpha} \in (-1, 1)$ such that the firm sets negative fee for all $\alpha < \dot{\alpha}$.*

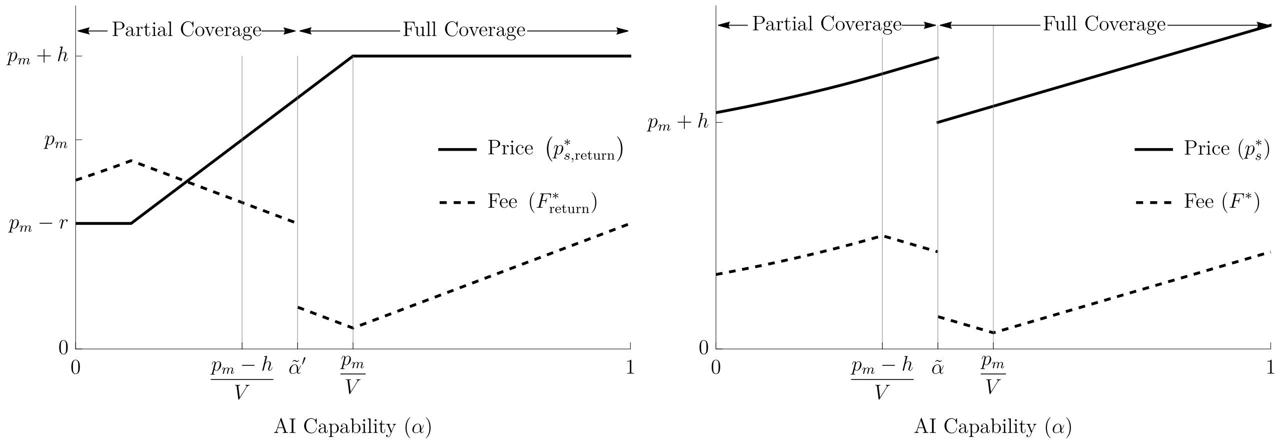
5.2 Firm Return Processing Cost

The main model assumed that only the consumers incurred hassle cost of returning products. In practice, however, the firm also incurs substantial costs from processing return products (e.g., Jerath and Ren, 2021). According to an executive at an operations firm that streamlines product returns, "processing online returns can cost \$10 to \$20, excluding freight" (The Wall Street Journal, 2021).[27]

In this section, we enrich the model by considering return processing costs for the firm. Whenever a consumer returns an unwanted product (at hassle cost h), the firm incurs an operational cost $r > 0$ for processing the return. Here, r parsimoniously captures various costs associated with handling returns such as storage, re-packaging, returning products to manufacturer, etc.

Our analysis reveals that return costs incentivize the firm to reduce product returns by lowering the product price. Interestingly, this downward pressure on price only applies if α is small. The

[27] `https://on.wsj.com/3y8giwW` (accessed July 17, 2022)



(a) Return Cost ($s_H = r = 0.1$)

(b) Fee as Credit ($s_H = 0.1$)

Figure 6: Impact of AI Capability on Price and Fee ($V = 1, p_m = 0.5, h = 0.1$)

reason is that if α is large, the firm is likely to ship a high-match product such that the return rate is low. On the other hand, if α is small, there is a high risk of product mismatch; in this case, the firm lowers the price to reduce the return rate (see Figure 6a). The following proposition summarizes the impact of return costs on the firm's optimal price and fee.

Proposition 4. *With return cost r, the firm's equilibrium strategy $\left(p^*_{s,return}, F^*_{return}\right)$ is as follows:*

$$p^*_{s,return} = \min\left\{\max\left\{\alpha V + h, p_m - r\right\}, p_m + h\right\}, \text{ and } F^*_{return} = F^*_{part}(p^*_{s,return}) - \begin{cases} 0 & \text{if } \alpha \le \tilde{\alpha}', \\ s_H & \text{if } \alpha > \tilde{\alpha}', \end{cases}$$

where $F^*_{part}(\cdot)$ is from (4.2) and the threshold $\tilde{\alpha}'$ is as defined in the proof.

5.3 Fee as Credit Towards Purchase

Several ship-then-shop firms have implemented a pricing strategy that allows the service fee to be credited towards purchase. For example, Stitch Fix's \$20 styling fee, which is paid upfront, may be "credited toward any pieces [the customers] keep."[28] Similarly, Wantable claims that the \$20 styling fee "will be credited towards items [the customers] keep."[29] Based on this observation, we analyze a fee-as-credit pricing strategy and assess the robustness of the insights from the main model. If the firm sets fee F and product price p_s, and if the ship-then-shop subscriber decides to purchase the shipped product, then she pays $p_s - F$, as opposed to p_s as in the main model. All other model specifications remain the same.

[28] `https://www.stitchfix.com/pricing` (accessed July 17, 2022).

[29] `https://www.wantable.com/how-wantable-works/` (accessed July 17, 2022).

We demonstrate that allowing upfront fee to be credited towards purchase does not qualitatively change the core insights. Under the fee-as-credit regime, we find that compared to the baseline case without such credit policy, the equilibrium fee remains the same and the product price increases exactly by the fee amount. The intuition is that from a rational consumer's perspective, paying p_s vs. paying $p_s + F$ with credit F for the product she decides to keep yield the same utilities. Therefore, the only quantitative departure from the main model is that the product price increases by the upfront fee (see Figure 6b). The following proposition states this result.

Proposition 5. *The firm's optimal product price and subscription fee, when the fee can be credited towards product purchase, are* $F^*_{credit} = F^*$ *and* $p^*_{s,credit} = p^*_s + F^*$*, respectively, where* p^*_s *and* F^* *are, respectively, the optimal price and fee from* (18) *in the main model.*

5.4 Other extensions

We analyze additional extensions to further assess the robustness of our results. Overall, we find that the qualitative insights from the main model carry over. The following is an overview of those robustness checks, the detailed analyses of which can be found in the Online Appendix.

1. **Wholesale Price Discount** In the main model, we assumed that both the ship-then-shop firm and consumers face the same price at the traditional market. We relax this assumption and consider the possibility that the ship-then-shop firm may procure products from the traditional market at a lower price than do consumers (e.g., due to power in the distribution channel, volume discounts, etc.).
2. **Learning Dynamics** We explore a scenario in which the ship-then-shop firm can learn consumers' preferences from product returns. Specifically, if a subscriber returns a shipped product, she receives another product of higher match quality than the returned product. In practice, the ship-then-shop firm may use product returns (among others such as "style quiz," product reviews, etc.) as a form of customer feedback to learn about the subscriber's product preference. As a result, after the firm has received the returned item, the firm may curate a product of higher match quality.
3. **Unobservable Product Price** The main model assumes that the firm can commit to a product price that consumers can observe prior to their subscription decision (Jing, 2018; Mehra et al., 2018; Shin, 2007). While this is consistent with how a number of firms set prices in practice, there are cases in which firms do not price-commit. For example, firms that offer ship-then-shop may first collect subscription fee and then decide product price as they ship

the products to their subscribers (e.g., through hidden fees, surcharges, etc.). We assess the robustness of our main insights to relaxing the price-commitment assumption. Specifically, we delay the firm's product price decision from Stage 1 to Stage 3, which is when consumers receive the product and decide whether to purchase the shipped product.

4. **No showrooming case:** We also examine the case when there is no consumer showrooming to highlight the effect of showrooming.
5. **General $s_L \in (0, s_H)$:** Finally, we show that normalizing s_L to 0 is without loss of generality.

6 Conclusion

Advances in AI technology, driven by machine learning techniques and data digitization, are fundamentally reshaping the business landscape. As improvements in AI algorithms enable firms to predict consumer preferences with greater accuracy, firms are adapting by reinventing their core business strategy. A notable example of a business transformation gaining traction in the retail sector is the ship-then-shop program. Unlike the traditional shopping model, which begins with consumer search and ends with product shipment, under the ship-then-shop model, the firm leverages its AI capability to predict consumers' preferences and ships the product to them; consumers then evaluate product fit and decide whether to purchase or return the product. In this paper, we develop a parsimonious game theory model that unveils nuanced economic forces underlying the ship-then-shop subscription model in the presence of consumer showrooming incentives.

We show that the firm's optimal service fee and product price depend crucially on the trade-off between *ex-ante* and *ex-post* surplus extraction strategies. If the firm's prediction capability is sufficiently low, the firm capitalizes on the convenience effect (stemming from the reduction in consumer search cost): it raises the fee and lowers the price. This strategy emphasizes *ex-ante* surplus extraction. As the firm's AI capability improves to an intermediate range, the firm shifts weight to the matching effect (stemming from superior product fit) by adopting the *ex-post* surplus extraction strategies; i.e., lowering fee and raising price. However, when the firm's prediction capability advances further, the firm reverts to *ex-ante* surplus extraction due to prevent consumers from showrooming. Thus, consumers' incentive to showroom disciplines the firm by exerting downward pressure on price.

We find that the ship-then-shop subscription model is most profitable *(i)* when AI capability is advanced, *(ii)* when the search friction in the market is severe, or *(iii)* when the product match potential is large. We also show that the marginal return of AI capability on the firm's profit decreases in search friction but increases in product match potential. These insights provide potentially valuable guidance for managers implementing the innovative subscription model. For example, investing in AI capability is more fruitful when the product match potential is large, as it enables the firm to better reap the upside gains from the matching effect.

Taken together, we shed light on novel insights that have important managerial implications for ship-then-shop subscription models. In particular, our analysis highlights the significance of balancing the *ex-ante* vs. *ex-post* extraction strategies. Depending on the state of the firm's prediction capability and various market parameters, notably the degree of search friction in the market, the firm's optimal response to changes in prediction capability can vary qualitatively.

We acknowledge several limitations of the current study and discuss avenues for future research. First, the main model assumes that the utilities of ship-then-shop subscribers derive primarily from the product's match value and price. Enriching the consumer utility model by accounting for consumers' desire for product variety and surprise would be an exciting avenue for future research. Second, to keep the analysis tractable, we simplify the consumer-product match process as a single random draw, thereby excluding consumer's sequential search under repeated interactions. It would be interesting to relax this assumption and explore the strategic interaction between consumer search in the traditional market and the firm's pricing strategy. Finally, we assume that the hassle cost of returning products and the return processing cost for the firm were exogenously fixed. Another fruitful avenue for future research would be to consider (a) firm's actions that endogenously reduce consumers' hassle cost (e.g., the firm offers free returns, pick-up services for returned boxes, etc.), and (b) return processing costs that decrease in the firm's AI capability (due to AI-based efficiency improvements in operation and supply chain management).

Appendix

Proof of Lemma 1

Let consumers discount future payoffs by δ. Consumers' utility from subscribing to ship-then-shop is $u_s = \frac{\delta\left(\int_{\alpha V}^{\max\{\alpha V, p_s-h\}}(0-h)\,dv_s + \int_{\max\{\alpha V, p_s-h\}}^{V}(v_s-p_s)\,dv_s\right)}{V(1-\alpha)} = \frac{\delta(V(2\alpha h-2p_s+V)-\max\{\alpha V,p_s-h\}(\max\{\alpha V,p_s-h\}+2h-2p_s))}{2V(1-\alpha)}$. Therefore, $F^*_{\text{part}}(p_s) = u_s - \left(\frac{(V-p_m)^2}{2V} - s_H\right)$. Substituting $F^*_{\text{part}}(p_s)$ into the firm's profit expression under partial coverage (13) and differentiating with respect to p_s yields $\frac{\partial}{\partial p_s}\mathbb{E}[\pi_{\text{part}}] = \frac{1-\delta}{2}$ if $p_s < \alpha V + h$, and $\frac{\partial}{\partial p_s}\mathbb{E}[\pi_{\text{part}}] = \frac{(1-\delta)h+p_m-(2-\delta)p_s+(1-\delta)V}{2V(1-\alpha)}$ if $\alpha V + h \leq p_s$. FOC, combined with the showroom-prevention constraint $p_s \leq p_m + h$, implies $p^*_s = \min\left\{p_m + h, \max\left\{\alpha V + h, V + h - \frac{V+h-p_m}{2-\delta}\right\}\right\}$. Setting $\delta \uparrow 1$ yields $p^*_s = \min\{p_m + h, \max\{\alpha V + h, p_m\}\}$.

$$\text{Therefore, } F^*_{\text{part}} = s_H + \begin{cases} \frac{1}{2}\left(\alpha V - \frac{p_m^2}{V}\right) - h & \text{if } p_m \leq \alpha V, \\ p_m - \frac{p_m^2}{2V} - \frac{\alpha V}{2} - h & \text{if } \alpha V < p_m \leq \alpha V + h, \\ \frac{h^2-2h(p_m-\alpha V)+\alpha(V-p_m)^2}{2V(1-\alpha)} & \text{if } \alpha V + h < p_m. \end{cases}$$

Under full coverage, F is lowered by s_H such that low-type consumers' IC constraint bind. ■

Proof of Lemma 2

It suffices to show $\frac{\partial}{\partial p_s}F^*_{\text{part}}(p_s) < 0$. If $p_s \leq V + h$, $\frac{\partial}{\partial p_s}F^*_{\text{part}}(p_s) = -1$ when $p_s \leq \alpha V + h$, and $\frac{\partial}{\partial p_s}F^*_{\text{part}}(p_s) = -\frac{V+h-p_s}{V(1-\alpha)} < 0$, when $p_s > \alpha V + h$. ■

Proof of Lemma 3

First, the difference $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}]$ is increasing in α because $\frac{\partial\left(\mathbb{E}[\pi_{\text{full}}]-\mathbb{E}[\pi_{\text{part}}]\right)}{\partial\alpha} = \frac{(V+h-p_m)^2}{16V(1-\alpha)^2}$ if $\alpha \leq \frac{p_m-h}{V}$, and $\frac{\partial\left(\mathbb{E}[\pi_{\text{full}}]-\mathbb{E}[\pi_{\text{part}}]\right)}{\partial\alpha} = \frac{V}{4}$ if $\alpha > \frac{p_m-h}{V}$. Second, $(\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}])\,|_{\alpha=0} < 0$ and $\lim_{\alpha\uparrow 1}(\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}]) > 0$, because $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}]|_{\alpha=0} = \frac{h^2-2hp_m-2s_HV}{4V} \leq \frac{h^2-2hp_m-2s_HV}{4V}|_{h=0} = -\frac{s_H}{2} < 0$, and $\lim_{\alpha\uparrow 1}\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] = \frac{V^2-2s_HV-p_m^2}{4V} \geq \frac{V^2-2s_HV-p_m^2}{4V}|_{s_H=\frac{(V-p_m)^2}{2V}} = \frac{(V-p_m)p_m}{2V} > 0$. Finally, Intermediate Value Theorem (IVT) ensures unique existence of $\tilde{\alpha} \equiv \{\alpha \in (0,1) : \mathbb{E}[\pi_{\text{full}}] = \mathbb{E}[\pi_{\text{part}}]\}$, such that $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0 \Leftrightarrow \alpha \geq \tilde{\alpha}$. Next, we show $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0$ is equivalent to s_H being small. First, algebraic manipulations yield $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0 \Leftrightarrow \frac{1}{2}\left(F^*_{\text{part}}(p^*_s) + \int_{\bar{v}}^{V}\frac{p^*_s-p_m}{V(1-\alpha)}dv_s\right) - s_H \geq 0$. Note that $\frac{\partial}{\partial s_H}\left(\frac{1}{2}\left(F^*_{\text{part}}(p^*_s) + \int_{\bar{v}}^{V}\frac{p^*_s-p_m}{V(1-\alpha)}dv_s\right) - s_H\right) = -\frac{1}{2}$.

Second, if $s_H \downarrow 0$, then $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0$ because increasing F infinitesimally and excluding s_L-consumers is unprofitable. Let $\xi \equiv \frac{1}{2}\left(F^*_{\text{part}}(p^*_s) + \int_{\bar{v}}^{V} \frac{p^*_s - p_m}{V(1-\alpha)dv}\right) - s_H\big|_{s_H = \frac{(V-p_m)^2}{2V}}$. If $\xi \geq 0$, full coverage dominates partial coverage for all s_H. If $\xi < 0$, IVT ensures unique existence of $\hat{s} \in \left(0, \frac{(V-p_m)^2}{2V}\right)$ such that $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0$ iff $s_H \leq \hat{s}$. For general ξ, we have $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0 \Leftrightarrow \tilde{s} \equiv \min\left\{\hat{s}, \frac{(V-p_m)^2}{2V}\right\}$. This completes the proof. ■

Proof of Proposition 1

Comparative statics for p^*_s is trivial and omitted. Note $\frac{\partial F^*}{\partial \alpha} = \frac{(V+h-p_m)^2}{2V(1-\alpha)^2}$ if $\alpha \leq \frac{p_m - h}{V}$, $\frac{\partial F^*}{\partial \alpha} = -\frac{V}{2}$ if $\frac{p_m-h}{V} < \alpha \leq \frac{p_m}{V}$, $\frac{\partial F^*}{\partial \alpha} = \frac{V}{2}$ if $\frac{p_m}{V} < \alpha$. At $\alpha = \tilde{\alpha}$, coverage shifts from partial to full, such that F^* decreases by magnitude s_H. For comparative statics w.r.t. s_H, note F^* is independent of s_H under full coverage. Under partial coverage, $F^* = s_H + \zeta$, where ζ is independent of s_H. Lemma 3 implies full coverage for $s_H \leq \tilde{s}$ and partial coverage for $s_H > \tilde{s}$. Therefore, F^* is independent of s_H for $s_H \leq \tilde{s}$, increases by s_H at $s = \tilde{s}$, and increases for $s_H > \tilde{s}$. With respect to h, we have $\frac{\partial F^*}{\partial h} = -\frac{p_m - h - \alpha V}{V(1-\alpha)}$ if $\alpha \leq \frac{p_m-h}{V}$, and $\frac{\partial F^*}{\partial h} = -1$ if $\frac{p_m-h}{V} < \alpha$. ■

Proof of Lemma 4

The profit change with respect to α is as follows. If $\alpha \leq \frac{p_m-h}{V}$, then $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} = \frac{(V+h-p_m)^2}{2V(1-\alpha)^2} \cdot \begin{cases} 1, & s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V-p_m)^2}{2V(1-\alpha)}, \\ \frac{1}{2}, & s_H > \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V-p_m)^2}{2V(1-\alpha)} \end{cases} > 0$. Also, if $\frac{p_m-h}{V} < \alpha$, $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} = \frac{V}{2} \cdot \begin{cases} 1 & \text{if } s_H \leq \frac{\alpha V^2 - p_m^2}{2V}, \\ \frac{1}{2} & \text{if } s_H > \frac{\alpha V^2 - p_m^2}{2V} \end{cases} > 0.$
Since $\mathbb{E}[\pi^*]$ is continuous in α, this suffices to establish that $\mathbb{E}[\pi^*]$ increases in α. The profit change with respect to s_H is as follows. If $\alpha \leq \frac{p_m-h}{V}$, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0 & \text{if } s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V-p_m)^2}{2V(1-\alpha)}, \\ \frac{1}{2} & \text{if } s_H > \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V-p_m)^2}{2V(1-\alpha)} \end{cases} \geq 0;$

Also, if $\frac{p_m-h}{V} < \alpha$, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0 & \text{if } s_H \leq \frac{\alpha V^2 - p_m^2}{2V}, \\ \frac{1}{2} & \text{if } s_H > \frac{\alpha V^2 - p_m^2}{2V} \end{cases} \geq 0$. Since $\mathbb{E}[\pi^*]$ is continuous in s_H, this suffices to establish that $\mathbb{E}[\pi^*]$ increases in s_H. The profit change with respect to V is as follows. If $\alpha \leq \frac{p_m-h}{V}$, then $\frac{\partial \mathbb{E}[\pi^*]}{\partial V} = \frac{2hp_m - h^2 + \alpha(V - p_m^2)}{2V^2(1-\alpha)} \cdot \begin{cases} \frac{1}{2} & \text{if } s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(p_m - V)^2}{2V(1-\alpha)}, \\ 1 & \text{if } s_H > \frac{h^2 - 2h(p_m - \alpha V) + \alpha(p_m - V)^2}{2V(1-\alpha)}. \end{cases}$ Note that the numerator $2hp_m - h^2 + \alpha(V - p_m^2)$ is positive because $\frac{\partial}{\partial h}\left(2hp_m - h^2 + \alpha(V - p_m^2)\right) = 2(p_m - h) > 0$, which implies that $2hp_m - h^2 + \alpha(V - p_m^2) \geq 2hp_m - h^2 + \alpha(V - p_m^2)|_{h=0} = \alpha(V - p_m^2) > 0$. Therefore, $\frac{\partial \mathbb{E}[\pi^*]}{\partial V} > 0$. If $\frac{p_m - h}{V} < \alpha$, $\frac{\partial \mathbb{E}[\pi^*]}{\partial V} = \frac{1}{2}\left(\alpha + \frac{p_m^2}{V^2}\right) \cdot \begin{cases} \frac{1}{2} & \text{if } s_H \leq \frac{\alpha V^2 - p_m^2}{2V}, \\ 1 & \text{if } s_H > \frac{\alpha V^2 - p_m^2}{2V} \end{cases} > 0$.

The profit change with respect to h is as follows: $\frac{\partial \mathbb{E}[\pi^*]}{\partial h} = -\max\left\{0, \frac{(p_m - \alpha V) - h}{2V(1-\alpha)}\right\} \leq 0$. ■

Proof of Proposition 2

The value of $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ if s_H is greater than the thresholds in the proof of Lemma 4 is half the value of $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ if s_H is less than the thresholds. Therefore, the derivative $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ decreases in s_H. Next, $\frac{\partial}{\partial V}\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} > 0$ because $\frac{V}{2}$ and $\frac{(V+h-p_m)^2}{2V(1-\alpha)^2}$ both increase in V. Moreover, $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ increases at $V = \frac{p_m - h}{\alpha}$ due to Claim 2. Excluding discontinuities, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ is independent of V. Finally, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ decreases at discontinuities due to Claim 3. ■

Proof of Proposition 3

The expressions for the equilibrium price and fee are the same as in the main model because the analysis from the main model does not depend on the constraint $\alpha \geq 0$. Per Lemma 3, the firm chooses partial coverage and sets $p_s^* = p_m$ for $\alpha < 0$. Since $\frac{dF^*_{\text{part}}}{d\alpha} = \frac{(V+h-p_m)^2}{2V(1-\alpha)^2} > 0$, the equilibrium fee is increasing in α for $\alpha < 0$. Therefore, to derive the condition under which the equilibrium fee is negative, it suffices to consider when $F^*_{\text{part}}(\alpha = -1) < 0$. To that end, note that $F^*_{\text{part}}(\alpha = -1) = \frac{h^2 - 2h(p_m + V) - p_m^2 + 2p_m V + V(4s_H - V)}{4V}$ and $\frac{dF^*_{\text{part}}(\alpha=-1)}{dh} = -\frac{V + p_m - h}{2V} < 0$. Since $F^*_{\text{part}}(\alpha = -1)$ is decreasing in h, there exists $\dot{h}$ such that $F^*_{\text{part}}(\alpha = -1) < 0$ for all $h > \dot{h}$, in which case there exists a corresponding $\dot{\alpha} \in (-1, 0)$, such that $F^*_{\text{part}} < 0$ for all $\alpha < \dot{\alpha}$. ■

Proof of Proposition 4

The principal deviation from the main analysis is that whenever the consumer returns the unwanted product, the firm incurs cost r. The firm's expected profit (12) changes to $\mathbb{E}[\pi_{\text{return}}(p_s, F)] = N_s(p_s, F)\Big(F + \frac{\bar{v}}{V(1-\alpha)}(-r) + \frac{V-\bar{v}}{V(1-\alpha)}(p_s - p_m)\Big)$. We derive the optimal price $p^*_{s,\text{return}}$ and fee F^*_{return} by following the proof of Lemma 1. Let consumers discount future payoffs by δ. Consider the firm's expected profit from the product market (i.e., conditional on ship-then-shop subscription): $\pi_{\text{product}} =$

$\int_{\alpha V}^{\max\{\alpha V,p_s-h\}} \frac{1}{V(1-\alpha)}(-r)\,dv_s+\int_{\max\{\alpha V,p_s-h\}}^{V} \frac{1}{V(1-\alpha)}(p_s-p_m)\,dv_s = \frac{V(p_s-p_m+\alpha r)-(p_s-p_m+r)\max\{\alpha V,p_s-h\}}{(1-\alpha)V}$.
Thus, the expected profit under partial coverage is $\mathbb{E}[\pi_{\text{part}}] = \frac{1}{2}\left(F_{\text{part}} + \frac{V(p_s-p_m+\alpha r)-(p_s-p_m+r)\max\{\alpha V,p_s-h\}}{(1-\alpha)V}\right)$, where $F_{\text{part}} = \frac{\delta(V(2\alpha h-2p_s+V)-\max(\alpha V,p_s-h)(\max(\alpha V,p_s-h)+2h-2p_s))}{2(1-\alpha)V} - \frac{(V-p_m)^2}{2V}$. Differentiating this with respect to p_s yields $\frac{\partial}{\partial p_s}\mathbb{E}[\pi_{\text{part}}] = \begin{cases} \frac{1-\delta}{2} & \text{if } p_s < \alpha V + h, \\ \frac{(1-\delta)(V+h)+p_m-r-(2-\delta)p_s}{2(1-\alpha)V} & \text{if } \alpha V + h \le p_s. \end{cases}$

Since the profit is strictly increasing for all $p_s \le \alpha V + h$, the firm will always set $p_s \ge \alpha V + h$. If $p_s > \alpha V + h$, the profit is concave in p_s. FOC, combined with the showroom-prevention constraint $p_s \le p_m + h$, implies $p_s^* = \min\left\{p_m + h, \max\left\{\alpha V + h, \frac{(1-\delta)(V+h)+p_m-r}{2-\delta}\right\}\right\}$. Setting $\delta \uparrow 1$ yields $p_s^* = \min\{p_m + h, \max\{\alpha V + h, p_m - r\}\}$. Therefore,

$$F_{\text{part}}^* = s_H + \begin{cases} \frac{1}{2}\left(\alpha V - \frac{p_m^2}{V}\right) - h & \text{if } p_m \le \alpha V, \\ p_m - \frac{p_m^2}{2V} - \frac{\alpha V}{2} - h & \text{if } \alpha V < p_m \le \alpha V + h + r, \\ \frac{h^2-2p_m(h+r+\alpha V)+2hr+\alpha V(2h+V)+\alpha p_m^2+r^2+2rV}{2(1-\alpha)V} & \text{if } \alpha V + h + r < p_m. \end{cases}$$

Under full coverage, F is lowered by s_H such that low-type consumers' IC constraint binds. We obtain the optimal profits under partial and full coverage by substituting the above values, respectively, into the firm's expected profit under partial coverage, $\mathbb{E}[\pi_{\text{part}}]$ and into $\mathbb{E}[\pi_{\text{full}}] = F_{\text{part}} - s_H + \frac{V(p_s-p_m+\alpha r)-(p_s-p_m+r)\max\{\alpha V,p_s-h\}}{(1-\alpha)V}$.

Next, we show that there exists a unique threshold $\tilde{\alpha}'$ such that $\mathbb{E}[\pi_{\text{part}}] \ge \mathbb{E}[\pi_{\text{full}}]$ if $\alpha \le \tilde{\alpha}'$ and $\mathbb{E}[\pi_{\text{part}}] < \mathbb{E}[\pi_{\text{full}}]$ otherwise. To that end, consider the difference $\Delta = \mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}]$. Since the optimal price is the same under both regimes, and $F_{\text{part}}^* = F_{\text{full}}^* + s_H$, we have $\mathbb{E}[\pi_{\text{full}}] = F_{\text{full}}^* + \pi_{\text{product}} = 2\left(\frac{1}{2}\left(F_{\text{part}}^* + \pi_{\text{product}}\right) - \frac{1}{2}s_H\right) = 2\mathbb{E}[\pi_{\text{part}}] - s_H$, such that $\Delta = \mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] = \mathbb{E}[\pi_{\text{part}}] - s_H$. Since $\mathbb{E}[\pi_{\text{part}}]$ increases in α, so does Δ. Second, we show that $\Delta(\alpha = 0) < 0 < \Delta(\alpha \to 1)$ so that by IVT, a unique root exists where $\Delta = 0$. At $\alpha = 0$, we obtain $\Delta(\alpha = 0) = -\frac{1}{4V}\left(\max\{0, \min\{\max\{h, p_m - r\}, h + p_m\} - h\}^2 + 2(h - p_m + r)\max\{0, \min\{\max\{h, p_m - r\}, h + p_m\} - h\} + p_m^2 + 2s_H V\right)$, whose derivative with respect to p_m is $\frac{d\Delta(\alpha=0)}{dp_m} = \begin{cases} -\frac{p_m}{2V} & \text{if } p_m \le h + r, \\ -\frac{h+r}{2V} & \text{otherwise.} \end{cases}$ Since this derivative is negative and continuous in p_m, $\Delta(\alpha = 0)$ is decreasing in p_m; therefore, $\Delta(\alpha = 0) \le \Delta(\alpha = 0, p_m = 0) = -\frac{s_H}{2}$, which shows that $\Delta(\alpha = 0) < 0$. At $\alpha \to 1$, we obtain $\lim_{\alpha\to 1}\Delta = \frac{V^2-2s_H V-p_m^2}{4V}$. This is positive if and only if $p_m^2 \le V(V - 2s_H)$. But this holds for $s_H \le \frac{(V-p_m)^2}{2V}$ and $p_m \le V$ because $p_m \le V \Leftrightarrow p_m \le 2V - p_m \Leftrightarrow p_m^2 \le p_m(2V - p_m) \Leftrightarrow p_m^2 \le V\left(V - 2\frac{(V-p_m)^2}{2V}\right) \Rightarrow p_m^2 \le V\left(V - 2s_H\right)$, where the last ($\Rightarrow$) follows from the assumption that $s_H \leq \frac{(V-p_m)^2}{2V}$. In total, due to IVT, there exists a unique $\tilde{\alpha}' \in (0,1)$ such that $\Delta < 0$ if $\alpha < \tilde{\alpha}'$ and $\Delta > 0$ otherwise. ■

Proof of Proposition 5

Let $p_{s,\text{credit}}$ and F_{credit} denote the firm's product price and subscription fee when the fee is credited towards product purchase, and let p_s and F denote the price and fee without purchase credit. It suffices to show that the firm's profit for any $(p_{s,\text{credit}}, F_{\text{credit}})$ under the credit regime can be replicated by $(p_{s,\text{credit}} - F_{\text{credit}}, F_{\text{credit}})$ under the no-credit regime: if this holds, then $\mathbb{E}[\pi_{\text{credit}}(p_{s,\text{credit}}, F_{\text{credit}})] \leq \mathbb{E}[\pi(p_s, F)]$ by replicability, and $p^*_{s,\text{credit}} = p^*_s + F^*$ and $F^*_{\text{credit}} = F^*$ are the optimal price and fee because then $\mathbb{E}[\pi^*_{\text{credit}}] = \mathbb{E}[\pi^*]$. To that end, consider the consumers' payoffs under the two regimes. Under the credit regime, the product price $p_{s,\text{credit}}$ and the credit F_{credit} matter only if the consumer purchases the product, in which case the consumer's utility is $u_s = v_s - p_{s,\text{credit}} + F_{\text{credit}}$. This is equivalent to the consumer's utility when she purchases in the main model without the credit at price $p_s = p_{s,\text{credit}} - F_{\text{credit}}$: $u_s = v_s - (p_{s,\text{credit}} - F_{\text{credit}})$. Since these utilities are the same, at the point of deciding subscription, the consumer's expected utility from subscription under each regime is also the same if $F_{\text{credit}} = F$. This shows that consumers' payoffs and strategies across the two regimes are the same if

$$p_{s,\text{credit}} = p_s + F \text{ and } F_{\text{credit}} = F. \tag{A1}$$

Finally, if (A1) holds, the firm's payoff is also the same across the two regimes. To see this, if the subscriber does not purchase, the firm obtains 0 revenue, whereas if she purchases, then

$$\pi = \begin{cases} p_{s,\text{credit}} - p_m - F_{\text{credit}}, & \text{under the credit regime,} \\ p_s - p_m, & \text{under the no-credit regime.} \end{cases} \tag{A2}$$

The two payoffs in (A2) are equal if (A1) holds. ■

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Online Appendix for "Predictive Analytics and Ship-then-shop Subscription"

OA1 Other Extensions

OA1.1 Wholesale Price Discount

In the main model, we assumed that both the ship-then-shop firm and consumers face the same price at the traditional market. We relax this assumption and consider the possibility that the ship-then-shop firm may procure products from the traditional market at a lower price than do consumers (e.g., due to power in the distribution channel, volume discounts, etc.). To that end, suppose the firm can procure products at a price $p_m - \gamma$, for some $\gamma \in [0, p_m)$, whereas consumers purchase at price p_m.

Our analysis shows that the qualitative insights from the main model are robust to settings with lower wholesale prices (see Figure OA1). An interesting quantitative departure from the main model is the change in subscription coverage. Specifically, the parametric region for which the firm induces both consumer segments to subscribe becomes larger as the wholesale cost decreases. The intuition is simple: lower wholesale cost implies higher margin from product sales, which in turn increases the returns from the matching effect. As a result, for larger γ, the firm shifts from partial to full coverage for smaller α threshold in order to capitalize on the matching effect from the subscribers.

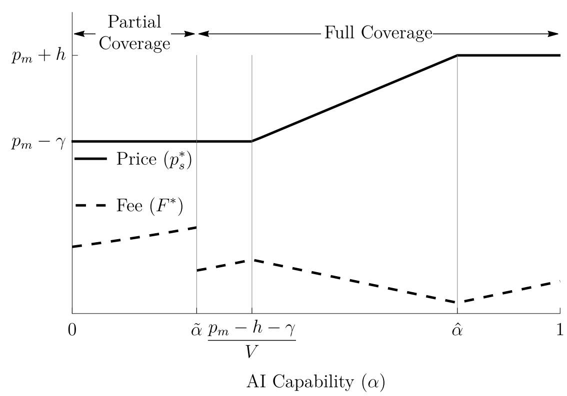

Figure OA1: Price and Fee with Wholesale Price Discount ($V = 1, p_m = 0.5, h = s_H = \gamma = 0.1$)

OA1.2 Learning Dynamics

In the main model, we assumed that if the ship-then-shop subscriber returns an unwanted product, she incurs hassle cost h and faces no further opportunity for product consumption. In practice, however, the ship-then-shop firm may use product returns as a form of customer feedback (among others such as "style quiz," product reviews, etc.) to learn about the subscriber's product preference.[1] As a result, after the firm has received the returned item, the firm may curate a product of higher match quality.[2] In this section, we explore a scenario where if a subscriber returns a shipped product,

[1]The firm may also learn about customer preferences from the non-returned items; however, this is less relevant to our setting as we assume that consumers purchase at most one unit.

[2]For example, Stitch Fix's algorithm leverages "clients' fit feedback and purchase histories" to learn customer styling preferences (`https://algorithms-tour.stitchfix.com/#recommendation-systems`).

she receives another product of higher match quality than the returned product.

The game extends the main model by considering a "product return subgame": after the consumer returns an unwanted product at hassle cost h, the firm ships to the consumer another product whose match value is higher than that of the previously returned product. To sharpen insights, we assume that the post-return product has the maximal match value $v_s' = V$. The firm sets product price p_s' for the post-return product. The consumer then decides whether to buy the product, return the product at hassle cost h, or showroom (i.e., buy the same product from the traditional market).

In the extended game, the ship-then-shop subscriber makes purchase decisions for potentially two separate products: the first product, which the subscriber receives upon service subscription, and the second product, which the subscriber receives if she returns the first product. We find that the firm charges $p_m + h$ for the second product, which is the maximum price that prevents consumers from showrooming. This is consistent with expectation because the firm identifies superior match quality for the second product based on the returned first product.

Perhaps more surprisingly, we show that the presence of learning increases the price for the first product relative to the main model, even though the match value distribution for the first product remains the same. Intuitively, learning serves as an insurance against the risk of product returns: while without learning, the firm obtained zero profit upon product return, with learning, the firm can capitalize on the upside of learning by shipping a second product of superior match quality. Thus, the firm charges a higher first product price to reap higher first-product margins from consumers who like the first product, while obtaining second-product profit from consumers who do not like the first product and return it. This upward pressure on the first product's price exerted by learning dynamics is most pronounced for smaller ranges of α where the risk of product returns is high. The following proposition summarizes this finding.

Proposition OA1. *The presence of learning dynamics increases the firm's product price if $\alpha < p_m/V$, and has no impact if $\alpha \geq p_m/V$.*

Proof of Proposition OA1. We begin by solving for the subgame in which the subscriber had returned the shipped product in the previous stage. In this last stage, the firm ships another product whose match value is V. Similar to the main model without learning, the firm will not set product price p_s greater than $p_m + h$. For then the subscriber will return the product (and potentially showroom in the traditional market). Therefore, $p_s \leq p_m + h$. Given this price cap, the subscriber will buy the shipped product in the last stage if and only if $V - p_s \geq -h$; i.e., $p_s \leq V + h$. Combined with the anti-showroom constraint $p_s \leq p_m + h$, we obtain

$$p_s^* = \min\{p_m + h, V + h\} = p_m + h,$$

where the second equality follows from the assumption that $p_m < V$.

In sum, the subscriber's expected utility in the last stage after returning the product in the previous stage is

$$\mathbb{E}[u_s] = V - (p_m + h), \tag{OA1}$$

and the firm's expected profit is

$$\mathbb{E}[\pi] = p_m + h - p_m = h. \tag{OA2}$$

With this subgame equilibrium at hand, we solve the Stage 3 game in which the consumer decides whether to (a) purchase the shipped product for utility $-F + v_s - p_s$, (b) return it at hassle cost h for utility $-F-h$ plus the subsequent utility of $V-(p_m+h)$ from (OA1), or (c) showroom at the traditional market for utility $-F-h+v_s-p_m$.

Therefore, the subscriber

(a) buys if $-F+v_s-p_s \geq -F-h+V-p_m-h$ and $-F+v_s-p_s \geq -F-h+v_s-p_m$, which simplifies to $p_s \leq p_m+h+\min\{v_s+h-V, 0\}$,
(b) returns if $-F-h+V-p_m-h \geq -F+v_s-p_s$ and $-F-h+V-p_m-h \geq -F-h+v_s-p_m$, which simplifies to $p_s \geq v_s+2h+p_m-V$ and $V-h \geq v_s$, and
(c) showroom if $-F-h+v_s-p_m > -F+v_s-p_s$ and $-F-h+v_s-p_m \geq -F-h+V-p_m-h$, which simplifies to $p_s > p_m+h$ and $v_s \geq V-h$.

We show that setting $p_s > p_m+h$ is dominated by $p_s = p_m+h$. If $p_s > p_m+h$, then either the subscriber returns or showrooms, depending on the realization of v_s, such that the firm's expected profit from the subscriber is

$$\mathbb{E}[\pi_1] = F_1 + \int_{\alpha V}^{\max\{\alpha V, V-h\}} \frac{h}{V(1-\alpha)}\, dv_s + \int_{\max\{\alpha V, V-h\}}^{V} \frac{0}{V(1-\alpha)}\, dv_s,$$

where the h in the first integral is the firm's expected future profit (OA2) from the product return subgame above.

On the other hand, if $p_s = p_m+h$, then either the subscriber returns or buy, depending on the realization of v_s, such that the firm's expected profit from the subscriber is

$$\mathbb{E}[\pi_2] = F_2 + \int_{\alpha V}^{\max\{\alpha V, V-h\}} \frac{h}{V(1-\alpha)}\, dv_s + \int_{\max\{\alpha V, V-h\}}^{V} \frac{(p_m+h)-p_m}{V(1-\alpha)}\, dv_s.$$

Recall that in equilibrium, the firm sets the subscription fee such that the consumer is indifferent between subscription and searching in the traditional market. Since consumers' expected utility from ship-then-shop subscription decreases in p_s, the firm is able to charge higher subscription fee under $p_s = p_m+h$ than under $p_s > p_m+h$; i.e., $F_2 \geq F_1$. Taken together, we obtain

$$\max_{p_s > p_m+h} \mathbb{E}[\pi_1] \leq \mathbb{E}[\pi_2]|_{p_s=p_m+h}.$$

Therefore, in equilibrium, the firm will only consider $p_s \leq p_m+h$, such that the outcome in which subscribers showroom is ruled out.

Next, we compute the optimal fee and price under $p_s \leq p_m+h$. The optimal fee (as a function of p_s) is obtained at the point where the subscriber is indifferent between subscribing and searching in the traditional market:

$$\begin{aligned}\mathbb{E}[u_s] &= -F + \int_{\alpha V}^{M(p_s)} \frac{-h+(V-p_m-h)}{V(1-\alpha)}\, dv_s + \int_{M(p_s)}^{V} \frac{v_s-p_s}{V(1-\alpha)}\, dv_s \\ &= -s + \frac{(V-p_m)^2}{2V} = \mathbb{E}[u_m] \text{ for either } s=s_L \text{ or } s=s_H,\end{aligned}$$

where $M(p_s) = \max\{\alpha V, V-2h-p_m+p_s\}$. Therefore,

$$\begin{aligned}F^*_{\text{part}}(p_s) &= \int_{\alpha V}^{M(p_s)} \frac{-h+(V-p_m-h)}{V(1-\alpha)}\, dv_s + \int_{M(p_s)}^{V} \frac{v_s-p_s}{V(1-\alpha)}\, dv_s - \frac{(V-p_m)^2}{2V} + s_H, \\ F^*_{\text{full}}(p_s) &= \int_{\alpha V}^{M(p_s)} \frac{-h+(V-p_m-h)}{V(1-\alpha)}\, dv_s + \int_{M(p_s)}^{V} \frac{v_s-p_s}{V(1-\alpha)}\, dv_s - \frac{(V-p_m)^2}{2V}.\end{aligned}$$

The firm's expected profit under partial coverage is

$$\mathbb{E}[\pi_{\text{part}}] = \frac{1}{2}\left(F^*_{\text{part}}(p_s) + \int_{\alpha V}^{M(p_s)} \frac{h}{V(1-\alpha)}\,dv_s + \int_{M(p_s)}^{V} \frac{p_s - p_m}{V(1-\alpha)}\,dv_s\right)$$
$$= \frac{1}{2}\left(\int_{\alpha V}^{M(p_s)} \frac{V - p_m - h}{V(1-\alpha)}\,dv_s + \int_{M(p_s)}^{V} \frac{v_s - p_m}{V(1-\alpha)}\,dv_s - \frac{(V-p_m)^2}{2V} + s_H\right),$$

and similarly, under full coverage, it is

$$\mathbb{E}[\pi_{\text{full}}] = \int_{\alpha V}^{M(p_s)} \frac{V - p_m - h}{V(1-\alpha)}\,dv_s + \int_{M(p_s)}^{V} \frac{v_s - p_m}{V(1-\alpha)}\,dv_s - \frac{(V-p_m)^2}{2V}.$$

In either regime, the optimal p_s is the same. Following the logic of asymptotic discounting in the proof of Lemma 1, we obtain that $\max\{p_m + h, \alpha V + h\}$ maximizes the firm's expected profit. Combined with the anti-showrooming constraint $p_s \le p_m + h$, we obtain

$$p_s^* = p_m + h.$$

Compared to the optimal price from the main model, which plateaus to $p_m + h$ at $\alpha = p_m/V$, the result follows. ■

OA1.3 Unobservable Product Price

The main model assumes that the firm can commit to a product price that consumers can observe prior to their subscription decision (Jing, 2018; Mehra et al., 2018; Shin, 2007). While this is consistent with how a number of firms set prices in practice, there are cases in which firms do not price-commit. For example, firms that offer ship-then-shop may first collect subscription fee and then decide product price as they ship the products to their subscribers (e.g., through hidden fees, surcharges, etc.). In this section, we assess the robustness of our main insights to relaxing the price-commitment assumption. Specifically, we delay the firm's product price decision from Stage 1 to Stage 3, which is when consumers receive the product and decide whether to purchase the shipped product. All other model specifications remain unchanged.

Similar to the main model, consumers make subscription decisions based on subscription fee and product price. The key difference is that consumers cannot observe the *actual* price; instead, they consider the *expected* product price p_s^e. In Stage 3, the firm decides p_s taking into account p_s^e. Note that once consumers subscribe to ship-then-shop, their expected price is immaterial to the firm's profit. Conditional on consumer subscription, the firm's product pricing problem in Stage 3 is

$$\max_{p_s \le p_m + h} \frac{V - \bar{v}}{V(1-\alpha)}(p_s - p_m),$$

where $\bar{v} = \max\{\alpha V, p_s - h\}$ as in (8), and the constraint $p_s \le p_m + h$ prevents consumer showrooming. This yields

$$\tilde{p}_s^* = \min\left\{\max\left\{\alpha V + h, \frac{V + h + p_m}{2}\right\}, p_m + h\right\}. \tag{OA3}$$

In equilibrium, consumers' expectations align with the firm's optimal price. Therefore, $\tilde{p}_s^*$ in (OA3) is the equilibrium price in the scenario without price-commitment. Barring a minor linear transformation, $\tilde{p}_s^*$ is identical to $p_s^* = \min\{\max\{\alpha V + h, p_m\}, p_m + h\}$, the optimal price with

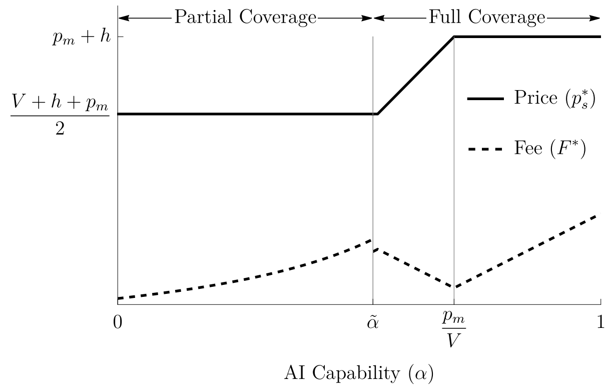

Figure OA2: Impact of AI Capability on Price and Fee ($V = 1, p_m = 0.75, h = 0.5, s_H = 0.02$)

price-commitment in the main model. In particular, all the comparative statics with respect to α, V, h, and p_m qualitatively carry over from the main model.

Furthermore, since the remaining model features are unchanged, the qualitative insights pertaining to consumers' subscription decisions and the firm's optimal fee sustain as well. For instance, the qualitative patterns of equilibrium price and fee patterns with respect to AI capability (α) are preserved. Figure OA2 shows the impact of AI capability on price p_s and fee F without price commitment (compare with Figure 4).

OA1.4 No Showrooming

In the main model, the consumers had incentive to showroom, which disciplined the ship-then-shop firm to cap the product price at $p_m + h$. In this section, we examine the firm's equilibrium price and fee strategy when showrooming effect is muted.

The analysis is largely the same as the main model, except that the anti-showrooming constraint $p_s \leq p_m + h$ is relaxed. Interestingly, if the firm's prediction accuracy is sufficiently high (i.e., $\alpha > \hat{\alpha}$), the subscription fee becomes negative (see Figure OA3). The firm entices consumers to join the ship-then-shop program by offering a "sign-up bonus," and then extracts the large *ex-post* surplus through high product price if there is no consumer showrooming.[3] This is in stark contrast to the main model with the potential threat of consumer showrooming.

Lemma OA1. *The firm's optimal subscription fee F^* is negative if $\alpha > \hat{\alpha}$, where $\hat{\alpha} \in \left[\frac{p_m - h}{V}, 1\right)$.*

Proof of Lemma OA1. Let $\tilde{\alpha}$ be threshold value defined in Lemma 3. Suppose $\alpha > \max\left[\tilde{\alpha}, \frac{p_m-h}{V}\right]$ such that $p^* = \alpha V + h$ and $\mathbb{E}[\pi_{\text{full}}] > \mathbb{E}[\pi_{\text{part}}]$. Note $\frac{\partial F^*}{\partial \alpha} = -\frac{V}{2} < 0$. At $\alpha = \frac{p_m - h}{V}$, F^* under full coverage attains maximum value, and $\lim_{\alpha \uparrow 1} F^* = -h - \frac{(V-p_m)^2}{2V} < 0$. Therefore, by the Intermediate Value Theorem, there exists a unique $\hat{\alpha} \in [(p_m - h)/V, 1)$ such that $F^* < 0$ for all $\alpha > \hat{\alpha}$. ■

[3] Note that the firm's incentive for offering sign-up bonus is subtly distinct from the case in which the firm subsidize purchase for negative (see Section 5.1). In contrast to the demand-generating role of sign-up bonuses in the no-showrooming case, the primary role of sign-up bonuses in the case of inferior prediction capability was to compensate consumers for the poor expected match quality.

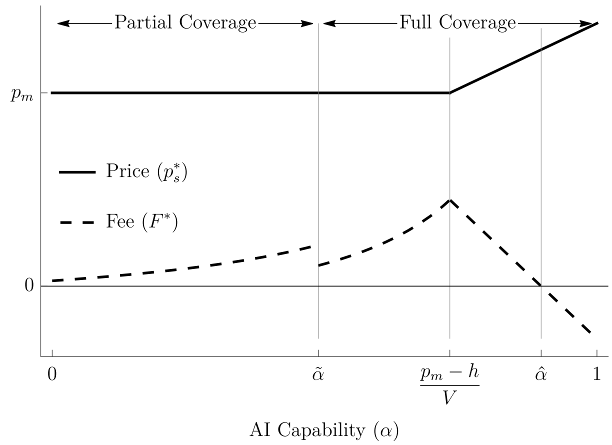

Figure OA3: Impact of AI Capability on Price and Fee ($V = 1, p_m = 0.75, h = s_H = 0.02$)

OA1.5 General $s_L \in (0, s_H)$

In this section, we show that normalizing s_L to 0 is indeed without loss of generality. It suffices to show that the insights under general $s_L \in (0, s_H)$ is qualitatively the same as that under $s_L = 0$ and some scaled $s_H > 0$. To that end, suppose $s_L \in (0, s_H)$.

First, from equation (13) and the fact that the optimal price p_s^* is independent of consumers' search costs, we obtain that $F^*_{\text{part}} = s_H + \xi_0$ and $F^*_{\text{full}} = s_L + \xi_0$ for some ξ_0 which is independent of s_L and s_H. Therefore, only departure from the normalized main model is that the optimal fee under full coverage is shifted upward by s_L. More generally, it follows that the qualitative insights pertaining to the dynamics of the optimal subscription fee and price with respect to the model primitives, in particular α, are preserved.

Second, since p_s^* is independent of coverage, the ship-then-shop firm's optimal profit can be compactly written as

$$\mathbb{E}[\pi^*] = \max\left\{\frac{1}{2}\left(s_H + \xi_1\right), s_L + \xi_1\right\},$$

for some ξ_1 which is independent of s_L and s_H. Therefore, the qualitative insights under general s_L can be mapped to the normalized main model as follows:

1. if $0 < s_L < \frac{1}{2}s_H$, then re-define search costs $s'_L = 0$ and $s'_H = s_H - 2s_L$; and
2. if $\frac{1}{2}s_H < s_L < s_H$, then full coverage dominates partial coverage (because $\frac{1}{2}\left(s_H + \xi_1\right) < s_L + \xi_1$); this is qualitatively the same as the case with $s'_L = 0$ and $s'_H = \epsilon$ for some small $\epsilon > 0$.

OA2 Statements and Proofs of Claims

Claim 1. *$p_s > p_m + h$ is dominated by setting $p_s \leq p_m + h$*

Proof. If $p_s > p_m + h$, the subscriber either returns the product or showrooms, depending on the realization of her match value; i.e., her expected utility from ship-then-shop subscription is

$$\mathbb{E}[u_s] = \int_{\alpha V}^{V} \frac{\max\{-h + v_s - p_m, 0 - h\}}{V(1-\alpha)}\, dv_s$$

On the other hand, if $p_s = p_m + h$, the subscriber either returns the product or purchases, depending on the realization of her match value; i.e., her expected utility from ship-then-shop subscription is

$$\begin{aligned}\mathbb{E}[u_s] &= \int_{\alpha V}^{V} \frac{\max\{v_s - p_s, 0 - h\}}{V(1-\alpha)}\, dv_s \\ &= \int_{\alpha V}^{V} \frac{\max\{v_s - (p_m + h), 0 - h\}}{V(1-\alpha)}\, dv_s.\end{aligned}$$

Since the optimal fee makes consumers indifferent between ship-then-shop subscription and shopping at the traditional market, the fact that the two utilities above are the same show that the optimal fee is the same in both cases. However, setting $p_s = p_m + h$ yields higher expected profit for the firm than $p_s > p_m + h$ because if $v_s > p_m$, consumers purchase from the firm at $p_s = p_m + h$, generating positive margin of h per sales, whereas they showroom for all $p_s > p_m + h$, generating 0 profit. Therefore,

$$\max_{p_s > p_m + h} \mathbb{E}[\pi] \leq \mathbb{E}[\pi]|_{p_s = p_m + h} \leq \max_{p_s \leq p_m + h} \mathbb{E}[\pi],$$

such that setting $p_s > p_m + h$ is dominated by setting $p_s \leq p_m + h$. ■

Claim 2. *$\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ is non-decreasing in V at $V = \frac{p_m - h}{\alpha}$.*

Proof. At $V = \frac{p_m - h}{\alpha}$, we have $\frac{V}{2} = \frac{(V+h-p_m)^2}{2V(1-\alpha)^2}$. Now, we obtain from the proof of Lemma 4 that $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ is decreasing in V at $V = \frac{p_m - h}{\alpha}$ only if $s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)}$ for $V < \frac{p_m - h}{\alpha}$ and $s_H > \frac{\alpha V^2 - p_m^2}{2V}$ for $V > \frac{p_m - h}{\alpha}$. Due to Claim 4 (see below), these conditions imply the conditions (OA4) and (OA5) (see below). However, the proof of Case (iii) in Claim 3 (see below) shows that (OA4) and (OA5) cannot jointly hold. This proves that $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ cannot decrease in V at $V = \frac{p_m - h}{\alpha}$. ■

Claim 3. *$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ decreases at discontinuities with respect to V.*

Proof. From the proof of Lemma 4, the profit change with respect to α: if $\alpha \leq \frac{p_m - h}{V}$, $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} = \frac{(V+h-p_m)^2}{2V(1-\alpha)^2} \begin{cases} 1, & s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V-p_m)^2}{2V(1-\alpha)}, \\ \frac{1}{2}, & s_H > \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V-p_m)^2}{2V(1-\alpha)}. \end{cases}$ Also, if $\frac{p_m - h}{V} < \alpha$, $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} = \frac{V}{2} \begin{cases} 1 & \text{if } s_H \leq \frac{\alpha V^2 - p_m^2}{2V}, \\ \frac{1}{2} & \text{if } s_H > \frac{\alpha V^2 - p_m^2}{2V}. \end{cases}$
Thus, there are three discontinuities to consider: V at which (i) $s_H = \frac{\alpha V^2 - p_m^2}{2V}$, (ii) $s_H = \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)}$, and (iii) $p_m \leq \alpha V + h$.
Consider Case (i), which applies to $p_m \leq \alpha V + h$, or equivalently $V \geq \frac{p_m - h}{\alpha}$. Due to Claim 4 (see below), if $s_H \leq \frac{\alpha V^2 - p_m^2}{2V}$, then $V \geq \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}$. Therefore, if $p_m \leq \alpha V + h$, or equivalently

$V \geq \frac{p_m - h}{\alpha}$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0 & \text{if } V \geq \frac{s+\sqrt{s^2-\alpha p_m^2}}{\alpha}, \\ \frac{1}{2} & \text{if } V < \frac{s+\sqrt{s^2-\alpha p_m^2}}{\alpha}. \end{cases}$$

Consider Case (ii), which applies to $p_m > \alpha V + h$, or equivalently $V < \frac{p_m - h}{\alpha}$. Due to Claim 4 (see below), the inequality $s_H \leq \frac{h^2-2h(p_m-\alpha V)+\alpha(V-p_m)^2}{2V(1-\alpha)}$ is equivalent to

$$V \geq \frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h-2p_m+s)\right)}}{\alpha}.$$

Therefore, if $p_m > \alpha V + h$, or if $V < \frac{p_m - h}{\alpha}$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0 & \text{if } V \geq \frac{s-\alpha(h-p_m+s)+\sqrt{(1-\alpha)(s^2-\alpha(h+s)(h-2p_m+s))}}{\alpha}, \\ \frac{1}{2} & \text{if } V < \frac{s-\alpha(h-p_m+s)+\sqrt{(1-\alpha)(s^2-\alpha(h+s)(h-2p_m+s))}}{\alpha}. \end{cases}$$

Consider Case (iii). $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ increases at discontinuity only if $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = 0$ for $p_m > \alpha V + h$ and $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \frac{1}{2}$ for $p_m < \alpha V + h$. This requires:

$$\frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h-2p_m+s)\right)}s}{\alpha} < \frac{p_m - h}{\alpha} \tag{OA4}$$

and

$$\frac{p_m - h}{\alpha} < \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}, \tag{OA5}$$

which simplify to

$$\frac{h(h+2s)}{2(h+s)} < p_m < \frac{h+s}{2} \text{ and } \alpha > \frac{s^2}{(h+s)(h-2p_m+s)}. \tag{OA6}$$

Since $\frac{\partial}{\partial p_m} \frac{s^2}{(h+s)(h-2p_m+s)} > 0$, (OA6) holds only if the inequality $\alpha > \frac{s^2}{(h+s)(h-2p_m+s)}$ holds for smallest value of p_m, which under condition $\frac{h(h+2s)}{2(h+s)} < p_m < \frac{h+s}{2}$ is $p_m = \frac{h(h+2s)}{2(h+s)}$. But

$$\frac{s^2}{(h+s)(h - 2\frac{h(h+2s)}{2(h+s)} + s)} = 1,$$

which contradicts $\alpha < 1$. ■

Claim 4. *$p_m \leq V$ implies the following two equivalences:*

$$s_H \leq \frac{\alpha V^2 - p_m^2}{2V} \Leftrightarrow V \geq V_1 \textit{ and } s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)} \Leftrightarrow V \geq V_2,$$

where

$$V_1 \equiv \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}$$

and

$$V_2 \equiv \frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)}}{\alpha}.$$

Proof. For the first equivalence, note that $s_H \leq \frac{\alpha V^2 - p_m^2}{2V}$ can be rearranged in terms of V to the condition that either $V \leq \frac{s - \sqrt{s^2 - \alpha p_m^2}}{\alpha}$ or $V \geq \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}$. We show that the first condition cannot hold if $p_m \leq V$. Since $p_m \leq V$, we have

$$V \leq \frac{s - \sqrt{s^2 - \alpha p_m^2}}{\alpha} \Rightarrow p_m \leq \frac{s - \sqrt{s^2 - \alpha p_m^2}}{\alpha} \Rightarrow -2s_H \geq p_m(1-\alpha),$$

which is impossible since s_H and p_m are positive and $\alpha < 1$.

For the second equivalence, note that $s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)}$ can be rearranged in terms of V to the condition that either $V \leq \frac{s - \alpha(h - p_m + s) - \zeta}{\alpha}$ or $V \geq \frac{s - \alpha(h - p_m + s) + \zeta}{\alpha}$, where $\zeta = \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)}$. We show that the first condition cannot hold if $p_m \leq V$. Since $p_m \leq V$, we have $V \leq \frac{s - \alpha(h - p_m + s) - \zeta}{\alpha}$ implies $p_m \leq \frac{s - \alpha(h - p_m + s) - \zeta}{\alpha}$, which simplifies to

$$s(1-\alpha) - \alpha h - \zeta \geq 0. \tag{OA7}$$

The left-hand side of (OA7) is decreasing in h due to Claim 5 (see below); therefore, the inequality (OA7) must hold at $h = 0$. But at $h = 0$, the left-hand side of (OA7) reduces to

$$(1-\alpha)s - \sqrt{(1-\alpha)s(2\alpha p_m + (1-\alpha)s)},$$

which is less than 0, because

$$\sqrt{(1-\alpha)s(2\alpha p_m + (1-\alpha)s)} \geq \sqrt{(1-\alpha)s(0 + (1-\alpha)s)} = (1-\alpha)s.$$

Therefore, (OA7) does not hold for any $h \geq 0$. ■

Claim 5. *If $\alpha \leq 1$, $0 \leq h \leq p_m$ and $s \geq 0$, then*

$$\frac{\partial}{\partial h}\left(s(1-\alpha) - \alpha h - \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)}\right) \leq 0. \tag{OA8}$$

Proof. Writing out the derivative and simplifying yields

$$\text{(OA8)} \Leftrightarrow \sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m - h - s) + s^2\right)} > (1-\alpha)(-(p_m - h - s)).$$

First, suppose $p_m \geq h + s$ such that $(1-\alpha)(-(p_m - h - s))$ is negative. Since

$$\sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m - h - s) + s^2\right)} > 0,$$

the inequality holds. Second, suppose $p_m < h + s$. Then

$$\frac{\partial(h+s)(2p_m - h - s)}{\partial h} = -2(h + s - p_m) < 0 \text{ and } \frac{\partial(1-\alpha)(-(p_m - h - s))}{\partial h} = 1 - \alpha > 0,$$

such that $\sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m - h - s) + s^2\right)}$ is decreasing in h while $(1-\alpha)(-(p_m - h - s))$ is increasing in h. Therefore, to show that $\sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m - h - s) + s^2\right)} > (1-\alpha)(-(p_m -$

$h-s)$), it suffices to show that the inequality holds for the largest value of h in the interval $[0, p_m]$ (see Footnote 18). Substituting $h = p_m$ into the inequality and simplifying yields

$$\sqrt{(1-\alpha)\left(\alpha(p_m - s)(p_m + s) + s^2\right)} > (1-\alpha)s.$$

Since $\sqrt{(1-\alpha)\left(\alpha(p_m - s)(p_m + s) + s^2\right)}$ is increasing in p_m, we obtain

$$\sqrt{(1-\alpha)\left(\alpha(p_m - s)(p_m + s) + s^2\right)} > \sqrt{(1-\alpha)\left(\alpha(0 - s)(0 + s) + s^2\right)} = (1-\alpha)s.$$

This completes the proof. ■

OA3 No consumer switching between ship-then-shop and traditional market

Consider an infinite-period model where consumers can decide at each period whether to search in the traditional market or to subscribe to ship-then-shop. To show that the qualitative insights pertaining to consumers' subscription choices carry over, it suffices to prove that in equilibrium, consumers never switch from searching in the traditional market to subscribing to ship-then-shop. To see this, suppose towards a contradiction that there exists a consumer whose optimal strategy is to search in the traditional market for the first T periods, where $T \geq 1$, and then subscribe to ship-then-shop at Period $T + 1$.

According to Weitzman (1979), the optimal search order is completely characterized by each option's "reservation value." Since the reservation value of ship-then-shop subscription is fixed, if the reservation values of "sequential boxes" in the traditional market are (weakly) increasing, the optimal search cannot include a switch from traditional market to ship-then-shop. Specifically, suppose the reservation value of ship-then-shop subscription is z_s and the reservation values of sequential searches in the traditional market are $z_m^{(1)} \leq z_m^{(2)} \leq ...$, where the numerical superscripts denote the order in which the shop-then-ship "box" was opened. If the posited search order were indeed optimal−i.e., shop-then-ship for the first T-periods and then switch to ship-then-shop at Period $T+1$−it must be that

$$z_s < z_m^{(T)} \leq z_m^{(T+1)} < z_s,$$

where the first inequality reflects the dominance of searching in the traditional market at Period T, the second inequality the (weakly) increasing property of shop-then-ship reservation values, and the third inequality the dominance of ship-then-shop subscription at Period $T + 1$. This cannot hold, which contradicts the claim that the above search order is optimal.

In sum, consistent with the main model, even in an infinite-horizon dynamic setting, consumers will not start searching in the traditional market and then switch to ship-then-shop subscription. Consumers will either search in the traditional market or subscribe to ship-then-shop (and potentially free-ride the shipped product at the traditional market).