



Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

SCIENCE @ DIRECT®

International Journal of Industrial Organization  
23 (2005) 341–354

International Journal of  
Industrial  
Organization

[www.elsevier.com/locate/econbase](http://www.elsevier.com/locate/econbase)

# Imperfect price discrimination in a vertical differentiation model

Qihong Liu<sup>a,1</sup>, Konstantinos Serfes<sup>b,\*</sup>

<sup>a</sup>*Department of Resource Economics, University of Massachusetts, 300B Stockbridge Hall,  
80 Campus Center Way, Amherst, MA 01003-9246, United States*

<sup>b</sup>*Department of Economics and International Business, Bennett S. LeBow College of Business, Drexel University,  
Matheson Hall, 32nd and Market Streets, Philadelphia, PA 19104, United States*

Received 17 March 2004; accepted 29 January 2005

Available online 1 April 2005

---

## Abstract

We explore the competitive implications of third-degree price discrimination based on consumer information of varying degrees of “precision” in a vertical differentiation duopoly model. We show that, if the cost of information is below a threshold, only the high quality firm will acquire it and offer targeted promotions, while the low quality firm will commit to a uniform price, for any degree of consumer information precision. Equilibrium profits of the high quality firm are monotonically increasing and that of the low quality firm monotonically decreasing as a function of the consumer information precision. Finally, social and consumer welfare are monotonically increasing with respect to the precision of consumer information.

© 2005 Elsevier B.V. All rights reserved.

*JEL classification:* D43; L13; O30

*Keywords:* Imperfect price discrimination; Information acquisition; Vertical differentiation

---

---

\* Corresponding author. Tel.: +1 215 895 6816; fax: +1 215 895 6975.

*E-mail addresses:* [qiliu@resecon.umass.edu](mailto:qiliu@resecon.umass.edu) (Q. Liu), [ks346@drexel.edu](mailto:ks346@drexel.edu) (K. Serfes).

<sup>1</sup> Tel.: +1 413 545 6650; fax: +1 413 545 5853.

## 1. Introduction

The rapid development of the Internet as a medium of communication and commerce has prompted many marketers and retailers to accumulate a large amount of customer-specific information. Firms analyze the available information with the aid of sophisticated software tools and segment their customers according to certain verifiable characteristics such as age, income, place of residence, browsing patterns and past purchasing behavior. Consumer segmentation, in turn, helps firms tailor their promotional strategies, depending upon each group's preferences for a firm's brand. For example, different segments can receive a different quality of service or different discounts.<sup>2</sup> How refined customer segmentation is, depends critically on the quality and quantity of customer-specific information that a firm has acquired. The continuing growth of information technology (IT) has clearly had a positive impact on the ability of customer databases to predict consumer preferences more accurately.

We formulate a model of oligopolistic third-degree price discrimination with one high and one low quality firm (vertical differentiation). Consumer information partitions the characteristics space, allowing firms to classify the consumers into different segments by imperfectly estimating the brand premium each consumer is willing to pay for the high quality product. Firms can then tailor their prices to each consumer segment. Higher information precision is modeled as a refinement of the partition. We address the following questions: (i) Does the high quality firm have the stronger incentive to acquire information and price discriminate? (ii) How does the precision of the available information affect product quality? (iii) How do firms' incentives for information acquisition, profits and welfare evolve as information precision improves?

The literature on oligopolistic third-degree price discrimination in location models can be roughly divided into two strands.<sup>3</sup> The first strand assumes symmetric firms with the ability to segment consumers either into two groups or perfectly.<sup>4</sup> A common thread in those papers is that profits under price discrimination fall short of those under a uniform pricing rule. The second strand deals with asymmetric firms but maintains the assumption that firms can classify the consumers either into two groups or perfectly.<sup>5</sup> Corts shows that in a duopoly model of vertical differentiation, price discrimination between two groups of consumers leads to lower profits for *both* firms. Thisse and Vives develop a perfect price discrimination model and assume that one firm has a cost advantage over the other. They show that, in a dominant strategy equilibrium, both firms will choose to price discriminate.

---

<sup>2</sup> There are many examples of firms that segment consumers into groups based on observable characteristics and offer a different price to each group. Dell, for example, follows this practice. According to the June 8, 2001 *Wall Street Journal*: "One day recently, the Dell Latitude L400 ultralight laptop was listed at \$2307 on the company's Web page catering to small businesses. On the Web page for sales to health-care companies, the same machine was listed at \$2228, or 3% less. For state and local governments, it was priced at \$2072.04, or 10% less than the price for small businesses".

<sup>3</sup> For a comprehensive survey on the price discrimination literature see Stole (2003).

<sup>4</sup> See, for example, Bester and Petrakis (1996), Chen (1997), Fudenberg and Tirole (2000) and Shaffer and Zhang (1995).

<sup>5</sup> See, for example, Anderson and de Palma (1988), Corts (1998), Lederer and Hurter (1986), Rao (1991), Shaffer and Zhang (2000, 2002) and Thisse and Vives (1988).

The more efficient firm may or may not become better off compared to the non-discriminatory outcome, while the less efficient firm becomes worse off. Shaffer and Zhang (2002) employ a perfect price discrimination model with vertically and horizontally differentiated products. They demonstrate that if the marginal cost of targeting individual consumers is low then both firms offer targeted promotions; if the cost is in an intermediate range, *only* the larger (high quality) firm offers targeted promotions; and if the cost is high no firm charges more than one price. Rao's results have a similar flavor. Two-group price discrimination with asymmetric firms is taken up in Shaffer and Zhang (2000). Liu and Serfes (2004) examine price competition between two symmetric firms (i.e., pure horizontal differentiation), where, as in the present paper, the level of segmentation depends directly on the underlying precision of customer information.<sup>6</sup>

The aim of the present paper is to propose and solve a unifying price discrimination model, with two asymmetric (vertically differentiated) firms, which bridges the gap between the two-group and perfect discrimination paradigms.

We show that if the (fixed) cost of information is below a certain threshold, then in the unique Nash equilibrium *only* the high quality firm acquires information and practices price discrimination. The low quality firm's best response is to credibly commit not to engage in any price discrimination, by not purchasing the customer database (e.g. no haggle, no hassle pricing adopted by Saturn dealers). Interestingly, these strategies constitute an equilibrium irrespective of how refined the information is. The benefits and losses from price discrimination are distributed unevenly between the two firms, with the high quality firm emerging as the only winner. More specifically, equilibrium profits of the high quality firm monotonically increase as the information becomes more refined. In contrast, the profits of the low quality firm monotonically decrease with an increase in the precision of consumer information. Social and consumer welfare are monotonically increasing with respect to the precision of consumer information.

Contrary to Corts, who focuses on two-group discrimination and shows that both firms become worse off with price discrimination, we find that when firms have the ability to segment the consumers into *more than two* groups, the high quality firm becomes better off. Our results demonstrate the robustness of the Shaffer and Zhang (2002) result, that only the high quality firm price discriminates, even if we do not have perfect price discrimination. Finally, our results are in direct contrast to Thisse and Vives, who show that both types of firms will price discriminate. This highlights the importance of the source of firm asymmetry. While Thisse and Vives focus on cost-based asymmetry, we focus on demand side asymmetry.

The rest of the paper is organized as follows. The model and the four-stage game is presented in Section 2. The game is analyzed in Section 3. Section 3 also contains the welfare analysis. We summarize in Section 4.

---

<sup>6</sup> In our setting, the model with symmetric firms cannot be obtained as a special case of the asymmetric firms model, as it is the case in Shaffer and Zhang (2002) and Thisse and Vives. This is because the presence of customer information of varying degrees of precision has already complicated the analysis significantly and the addition of one more parameter (i.e., the one that would allow us to parameterize the degree of firm asymmetry) would make the analysis intractable. Therefore, one has to solve the case of pure horizontal differentiation separately from the case of pure vertical differentiation. See Sections 3.2 and 3.4 of the present paper for a comparison between the results obtained here and those in Liu and Serfes.

## 2. The description of the model

Two firms—low quality L and a high quality H, indexed by  $i$ —sell competing brands to a continuum of consumers. Consumers differ in their tastes (or income), described by the parameter  $\theta$  which is uniformly distributed on the interval  $[0,1]$  with density 1 (Mussa and Rosen, 1978). The quality of firm H's product is denoted by  $s_H$  and that of firm L by  $s_L$ , with  $s_H \geq s_L \geq 0$ . Let  $p_H$  and  $p_L$  denote the prices that firms H and L charge, respectively. Each consumer buys at most one unit of the good. Consumers have the same indirect utility function which is described as follows,

$$U = \begin{cases} V + \theta s_i - p_i, & \text{if a consumer purchases firm } i\text{'s product} \\ 0, & \text{otherwise.} \end{cases}$$

We assume that  $V$  is sufficiently high, ensuring that the market will be covered.<sup>7</sup> We denote the difference in product qualities,  $s_H - s_L$ , by  $\Delta$  and the premium consumers are willing to pay for the high quality product by  $x = \theta \Delta$ . It then follows that the parameter  $x$  is uniformly distributed on the interval  $[0, \Delta]$  with density  $1/\Delta$ . Therefore, a consumer who is located at point  $\hat{x} \in [0, \Delta]$  will purchase firm H's brand if and only if  $\hat{x} \geq p_H - p_L$ . Also, we assume that the cost function is of the form,  $C_i = s_i^2/2$  (i.e., marginal costs are zero).<sup>8</sup>

Firms can develop or acquire a database which helps them segment the consumers into distinct groups based upon each consumer's relative brand loyalty. For example, firms may obtain information about each consumer's income bracket. Consumers in the same bracket are pooled together. This indicates the premium people are willing to pay for the high quality product. We assume that the information partitions the  $[0, \Delta]$  interval into  $N$  sub-intervals (indexed by  $m$ ,  $m=1, \dots, N$ ) of equal length. In this case, a firm can charge different prices ( $p_{im}$ ,  $i=H,L$  and  $m=1, \dots, N$ ) to different groups of consumers, through the use of targeted coupons. Arbitrage between consumer groups is not feasible.

We further assume that  $N=2^k$ , where  $k$  takes on all integer values,  $k=0,1,2,3,4, \dots$ . Hence,  $N$  will parameterize the precision of consumer information, with higher  $N$ 's being associated with higher information precision. Moreover,  $N=1$  corresponds to no price discrimination and  $N=\infty$  to perfect price discrimination. Note that  $N$  does not take on all integer values, but rather  $N=1,2,4,8,16, \dots$  (i.e., information refinement). We assume that information of precision  $N$  is available to both firms at an exogenously given price (fixed cost)  $F > 0$  and that the current state of technology dictates  $N$  which the firms take as exogenously given. Hence, our model is static and the effect of information improvements on the equilibrium is captured by comparative statics analysis.

<sup>7</sup> See footnote 14 where we discuss how our results are affected when the market is allowed to be uncovered by setting  $V=0$ .

<sup>8</sup> Hence, we view the cost of developing a certain quality as a fixed cost (i.e., the cost of designing a product, or developing a brand name), which has no effect on the marginal cost of production [this assumption has been made by a number of papers in the vertical differentiation literature, e.g. Section II of Motta (1993)]. The analysis becomes intractable if we introduce a marginal cost of production that depends on the quality of the product.

We analyze a four-stage game with simultaneous and independent decisions at each stage. The game unfolds as follows:

- Stage 1: *Product quality decisions*. Firms choose  $s_H$  and  $s_L$ .
- Stage 2: *Information acquisition decisions*. Given the level of information precision  $N$  and acquisition cost  $F > 0$ , firms decide whether to acquire information.
- Stage 3: *Regular pricing decisions*. Firms choose their regular prices.
- Stage 4: *Promotional pricing decisions*. The firm(s) with information distribute(s) targeted price promotions (discounts) to the consumer segments.

Our setup parallels the multistage games that have been examined in the literature (e.g. Banks and Moorthy, 1999; Liu and Serfes, Rao, Shaffer and Zhang, 1995, 2002) and Thisse and Vives) where firms choose their promotional strategies after they have chosen their regular prices. This assumption is consistent with the common view that a firm's regular price can be adjusted more slowly than the value of targeted coupons. In addition, if both decisions are made simultaneously no pure strategy equilibrium exists in the subgames where only one firm has information (see also Shaffer and Zhang, (1995, footnote 11)).

In the next section, we look for a subgame perfect equilibrium of this game.

### 3. Analysis

We solve the game backwards, by first analyzing the four subgames after the firms' information acquisition decisions in stage 2. The first subgame is when neither firm acquires information and they both set their regular prices in stage 3. Stage 4 is never reached in this case. The second subgame occurs after both firms have acquired information. In stage 3, they both choose their regular prices and promotions take place in stage 4. In the third subgame only firm H possesses information, while in subgame four firm L is the sole owner of customer information. In the last two subgames, firms set their regular prices in stage 3 and the firm which has consumer information offers price discounts (in stage 4), after it observes the regular price of its rival. After having solved all four subgames, we proceed to stage 2 where the firms choose whether to acquire information or not. The cost of information,  $F$ , is ignored when we solve stages 3 and 4 of the game. It is re-introduced in stage 2. Finally, in stage 1, firms choose their product qualities.

#### 3.1. Pricing decisions (stages 3 and 4)

In this subsection, we solve for the equilibrium in each of the four subgames. The product qualities  $s_H$  and  $s_L$  (and consequently  $\Delta = s_H - s_L$ ) are fixed.

##### 3.1.1. Subgame 1: Neither firm has information (NI,NI)

The indifferent consumer is located at  $\hat{x} = p_H - p_L$ . Firms' demand functions are

$$d_H = 1 - \frac{(p_H - p_L)}{\Delta} \quad \text{and} \quad d_L = \frac{p_H - p_L}{\Delta}.$$

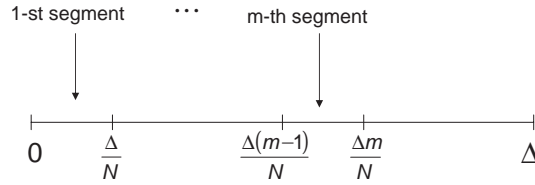


Fig. 1. Partition of the  $[0, \Delta]$  interval.

It can be easily shown that the equilibrium prices and profits are,

$$(p_H, p_L) = \left( \frac{2\Delta}{3}, \frac{\Delta}{3} \right) \text{ and } (\pi_H^{NI,NI}, \pi_L^{NI,NI}) = \left( \frac{4\Delta}{9}, \frac{\Delta}{9} \right) \tag{1}$$

3.1.2. Subgame 2: Both firms have information (I,I)

Without any loss of generality, we can ignore stage 3 and move directly to stage 4. When firms have the ability to change their prices in stage 4, stage 3 becomes irrelevant. Since both firms have acquired information, they know in which of the  $N$  segments each consumer is located and therefore they are able to charge different prices for different segments. The interval  $[0, \Delta]$  is equally divided into  $N$  segments, each one having length equal to  $\Delta/N$ . Segment  $m$  can be expressed as the interval  $[\Delta(m-1)/N, \Delta m/N]$ , where  $m$  is an integer between 1 and  $N$  (see Fig. 1).

In segment  $m$ , firms H and L charge prices  $p_{Hm}$  and  $p_{Lm}$ , respectively, the demands of their products are,

$$d_{Hm} = \frac{m}{N} - \frac{(p_{Hm} - p_{Lm})}{\Delta} \text{ and } d_{Lm} = \frac{(p_{Hm} - p_{Lm})}{\Delta} - \frac{(m-1)}{N},$$

with  $d_{Hm}$  and  $d_{Lm}$  in  $[0, 1/N]$ . Their profits are,

$$\pi_{Hm}(p_{Hm}, p_{Lm}) = p_{Hm}d_{Hm}, \text{ and } \pi_{Lm}(p_{Hm}, p_{Lm}) = p_{Lm}d_{Lm}.$$

Firm  $i$ 's, problem is,

$$\max_{p_{im} \geq 0} \pi_{im}(p_{Hm}, p_{Lm}), \text{ for each } m, m = 1, \dots, N, \text{ and } i = H, L.$$

The next proposition summarizes the solution to the above problem.

**Proposition 1.** Assume that both firms acquire information. Both firms sell positive quantities in the first segment ( $m=1$ ) for any  $N \geq 2$ . The equilibrium prices in the first segment are,

$$(p_{H1}, p_{L1}) = \left( \frac{2\Delta}{3N}, \frac{\Delta}{3N} \right).$$

In the remaining segments firm L's market share is zero, in equilibrium. The equilibrium prices in these segments are,

$$(p_{Hm}, p_{Lm}) = \left( \frac{(m-1)\Delta}{N}, 0 \right), \text{ where } m = 2, \dots, N.$$

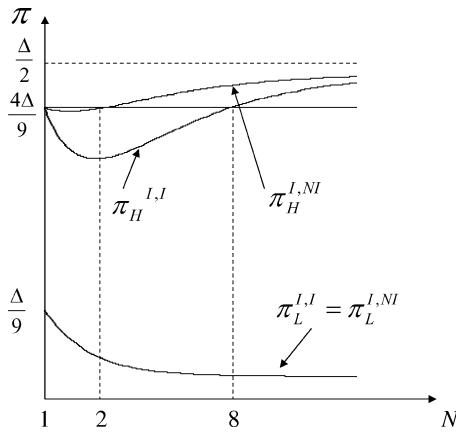


Fig. 2. Profits in subgames 2 and 3, (I,I) and (I,NI).

The equilibrium profits, over all segments, are,

$$\left( \pi_H^{I,I}(N), \pi_L^{I,I}(N) \right) = \left( \Delta \left[ \frac{9N^2 - 9N + 8}{18N^2} \right], \frac{\Delta}{9N^2} \right). \tag{2}$$

**Proof.** The proof is similar to the Proof of Proposition 1 in Liu and Serfes.<sup>9</sup> □

Firm H’s profits exhibit a U-shape as a function of the information precision  $N$ , exceed the non-discriminatory profits for  $N > 8$  and approach  $\Delta/2$  asymptotically. In contrast, firm L’s profits monotonically decrease with  $N$  and approach zero asymptotically (see Fig. 2).

The intuition behind the behavior of the equilibrium profits is as follows. In the standard vertical differentiation model, where each firm sets only one price (e.g. Shaked and Sutton, 1982), the high quality firm charges a relatively high price. This allows the low quality firm to realize a positive market share, despite the fact that all consumers prefer the high quality firm’s product. In our model, the availability of consumer information allows firms to segment the market and charge more than one price. This generates two opposing effects that govern market interaction. On the one hand competition intensifies because firms do not rank the different consumer segments in the same way (i.e., best response asymmetry, Corts). For instance, firm H’s strongest segment (i.e.,  $m = N$ ) is firm L’s weakest and vice versa. As a consequence a firm has a strong incentive to offer a deep discount to the relatively more loyal customers of the rival. The rival in response lowers its price and all-out competition ensues. On the other hand, firms can extract more consumer surplus from their loyal consumers. At low levels of information precision, the intensified competition effect dominates the surplus extraction effect. As the information becomes more refined, the high quality firm can extract more surplus from the consumers (and relatively more from those with high quality valuation than from those with lower quality valuation). This is how the U-shape of the high quality firm’s profit function is generated. The low quality firm, on the other hand, has no loyal customers and therefore its ability to extract surplus is diminished. Its price and market

<sup>9</sup> The proof can be found on the journal’s Web page.

share decrease as the high quality firm’s ability to target consumers increases. Not surprisingly, its profits approach zero asymptotically.

Firm asymmetry is critical for the results obtained in Proposition 1. With symmetric firms (i.e., when firms have equal customer bases at equal prices, as in a horizontal differentiation model), the surplus extraction effect is not as strong as it is when firms are asymmetric. In Liu and Serfes (where firm symmetry is assumed), the equilibrium profit functions of both firms are U-shaped and always below the non-discriminatory profits. Price competition is intense when no firm has a clear advantage. In contrast, in a vertical differentiation setting, the distribution of industry profits always favors the high quality firm.

Corts shows that when two vertically differentiated firms price discriminate between two consumer groups ( $N=2$  in our model), then price discrimination leads to lower profits for both firms (observation 2, p. 311). This is consistent with our model. If we set  $N=2$ , both firms are worse off relative to the no discrimination outcome. It can also be seen that focusing on two-group discrimination may be quite restrictive, as the high quality firm’s discriminatory profits quickly exceed (i.e., when  $N>8$ ) those under a uniform price (see Fig. 2).

3.1.3. Subgame 3: Only firm H (high quality firm) has information (I,NI)

We demonstrated in Proposition 1 that when both firms have acquired information, firm L has zero market share in all segments except the first one. It follows that when firm H has information and firm L does not, then firm L cannot sell to any customers other than the ones in the first segment. The reason is that when firm L charges a uniform price, that price has to be strictly positive, whereas when firm L has information (as in the I,I subgame) its price in all segments, other than the first one, is zero. Therefore, firm L will set, in stage 3, its uniform price to maximize its profits from the first segment only, ignoring the remaining customers. This price should be equal to firm L’s price in segment 1 ( $p_{L1}$ ) of subgame 2. It can be easily shown that the equilibrium prices in the first segment are,

$$(p_{H1}, p_L) = \left( \frac{2\Delta}{3N}, \frac{\Delta}{3N} \right).$$

Firm H, given firm L’s uniform price ( $p_L$ ), sets, in stage 4, its discriminatory prices in the remaining segments ( $m \geq 2$ ) to (just) drive firm L out of the market, i.e.,

$$d_{Lm} = \frac{(p_{Hm} - p_L)}{\Delta} - \frac{(m - 1)}{N} = 0 \Rightarrow p_{Hm} = \frac{\Delta(3m - 2)}{3N}, m = 2, \dots, N.$$

The equilibrium profits over all segments are,

$$\left( \pi_H^{I,NI}(N), \pi_L^{I,NI}(N) \right) = \left( \Delta \left[ \frac{9N^2 - 3N + 2}{18N^2} \right], \frac{\Delta}{9N^2} \right). \tag{3}$$

Firm H’s equilibrium profits monotonically increase with the precision of consumer information, while firm L’s profits monotonically decrease (Fig. 2). As in subgame 2, firm H prices firm L out of all consumer segments except the first one. The difference is that, in the present subgame, firm L does not fight for market share in the segments other than the first one. This is because firm L cannot charge more than one price and therefore it focuses



entirely on its stronger market (i.e., the first segment). As a consequence the surplus extraction effect is dominant for firm H in the remaining segments (i.e.,  $m \geq 2$ ) and its profits monotonically increase and dominate those from subgame 2 (i.e.  $\pi_H^{1,NI} > \pi_H^{1,I}$ ).

3.1.4. Subgame 4: Only firm L (low quality firm) has information (NI,I)

Firm H chooses its regular price  $p_H$  in stage 3 and firm L chooses its promotional prices  $p_{Lm}$ ,  $m=1, \dots, N$  in stage 4. We first solve firm L’s problem. Its demand in each segment is,

$$d_{Lm} = \frac{(p_H - p_{Lm})}{\Delta} - \frac{(m - 1)}{N}, \text{ for } m = 1, \dots, N.$$

Given  $p_H$  firm L chooses the depth of the discount that it offers to maximize,

$$\pi_L^{NI,I} = \sum_{m=1}^N p_{Lm} d_{Lm}.$$

Let  $p_{Lm}^*(N, p_H)$ ,  $m=1, \dots, N$  denote the solution to firm L’s maximization problem. Now let’s turn to firm H’s problem. Its demand in each segment is,

$$d_{Hm} = \frac{m}{N} - \frac{(p_H - p_{Lm}^*(N, p_H))}{\Delta}, \text{ for } m = 1, \dots, N.$$

Given the reaction function of firm L, firm H chooses its regular price  $p_H$  to maximize,

$$\pi_H^{NI,I} = p_H \sum_{m=1}^N d_{Hm}.$$

The next proposition summarizes the solution to the above problem. In the proof of this proposition we can no longer solve for the equilibrium in each segment separately, as we did in Proposition 1, because firm H due to its inability to charge more than one price cannot treat the consumer segments independently. This unbalanced distribution of information between the two firms adds to the difficulty and length of the proof significantly. Nonetheless, we were able to obtain closed form solutions and characterize the problem to its fullest extent.

**Proposition 2.** Assume that only firm L has acquired information. Then, for each  $N \geq 2$ , there exist two thresholds (integers)  $m_1$  and  $m_2$  (with  $0 \leq m_1 < m_2 \leq N + 1$ ) where,

$$m_1 = \frac{N}{2} - 1 \text{ and } m_2 = \frac{N}{2} + 2$$

such that:

- i) Firm H’s regular price is:  $p_H = \Delta(\frac{1}{2} + \frac{1}{4N})$ .
- ii) [This case is valid only when  $m_1 \geq 1$ ]. Firm L’s equilibrium demand is equal to  $1/N$  in all segments from 1 to  $m_1$ . Firm H’s equilibrium demand in these segments is zero. Moreover, firm L’s prices are,  $p_{Lm}^* = \Delta(\frac{1}{2} + \frac{1}{4N} - \frac{m}{N})$ ,  $m = 1, \dots, m_1$ .
- iii) Both firms sell positive quantities in the segments from  $m_1 + 1$  to  $m_2 - 1$ . Moreover, firm L’s prices are,  $p_{Lm}^* = \Delta(\frac{1}{4} + \frac{5}{8N} - \frac{m}{2N})$ ,  $m = m_1 + 1, \dots, m_2 - 1$ .

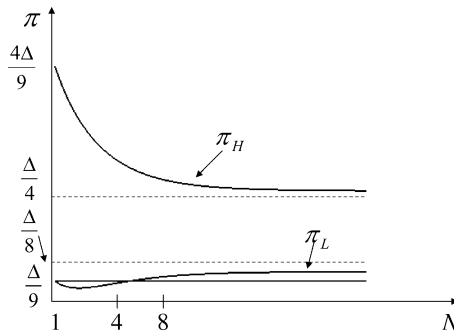


Fig. 3. Profits in subgame 4, (NI,I).

iv) [This case is valid only when  $m_2 \leq N$ ]. Firm H's equilibrium demand is equal to  $1/N$  in all segments from  $m_2$  to  $N$ . Firm L's equilibrium demand and prices in these segments are zero, i.e.,  $p_{Lm}^* = 0, m = m_2, \dots, N$ .

Finally, the equilibrium profits of each firm are,

$$(\pi_H^{NI,I}(N), \pi_L^{NI,I}(N)) = \left( \frac{\Delta(2N + 1)^2}{16N^2}, \frac{\Delta(4N^2 - 4N + 5)}{32N^2} \right). \tag{4}$$

**Proof.** The proof is similar to the Proof of Proposition 2 in Liu and Serfes.<sup>10</sup> □

Firm H's equilibrium profits monotonically decrease and approach  $\Delta/4$  as the information tends to become perfect. Firm L's profits on the other hand are U-shaped and approach  $\Delta/8$  asymptotically (Fig. 3). The intuition is similar to that after Proposition 1, with the only difference being that firm L has information while firm H does not. Therefore, firm H's initial advantage over firm L gradually diminishes as the precision of consumer information increases. Firm L is now able to extract consumer surplus after  $N$  exceeds a threshold, since its ability to target consumers increases and firm H cannot do much with only one price. Firm H's advantage, however, never disappears and that is why its profits are positive even when firm L has perfect information.

### 3.2. Information acquisition decisions (stage 2)

The game played between the two firms in the second stage can be summarized in Table 1 below.

The profits in the cells in Table 1 have been taken from Eqs. (1)–(4). We also include  $F > 0$ , the cost of acquiring information. The following proposition summarizes the equilibrium in the game.

<sup>10</sup> The proof can be found on the journal's Web page.

Table 1  
Stage 2: Firms' payoffs

H/L	NI	I
NI	$(\frac{4\Delta}{9}, \frac{\Delta}{9})$	$(\frac{\Delta(2N+1)^2}{16N^2}, \frac{\Delta(4N^2-4N+5)}{32N^2} - F)$
I	$(\Delta[\frac{9N^2-3N+2}{18N^2}] - F, \frac{\Delta}{9N^2})$	$(\Delta[\frac{9N^2-9N+8}{18N^2}] - F, \frac{\Delta}{9N^2} - F)$

**Proposition 3.**

1. If the cost of information,  $F$ , is below the threshold  $\bar{F}(N) = \frac{N^2-3N+2}{18N^2}$ , then only firm  $H$  (high quality firm) acquires information, i.e.,  $(I,NI)$  is the unique Nash equilibrium for all  $N \geq 2$ .
2. If the cost of information exceeds  $\bar{F}(N)$ , then neither firm acquires information, i.e.,  $(NI,NI)$  is the unique Nash equilibrium for all  $N \geq 2$ .

**Proof.** The proof is straightforward and is omitted. □

If the cost of information is relatively low (i.e., below  $\bar{F}(N)$ ), then the high quality firm enjoys the benefits of consumer information.<sup>11</sup> It finds it profitable to acquire information and becomes better off relative to the non-discriminatory outcome, even when the customer segmentation is not very refined. On the contrary, the low quality firm becomes worse off (see the equilibrium in subgame 3). Hence, the prisoners' dilemma, which is the recurrent outcome when firms are symmetric, is now avoided regardless of the precision of consumer information. The reason is that the price flexibility due to customer information enables the high quality firm to exploit the consumer preference bias in favor of its product. The low quality firm stands no chance when its rival can customize its prices. The low quality firm's best response is to commit to a uniform price, while the high quality firm engages in price discrimination. As a consequence, the high quality firm can extract consumer surplus without having to bear the adverse effects of intense price competition. What is quite interesting is that this behavior is an equilibrium outcome even when the quality of information is very low.<sup>12</sup>

Firm symmetry, as it is shown in Liu and Serfes, leads to a prisoners' dilemma when the precision of information exceeds a certain threshold (i.e.,  $N \geq 8$ ). When  $N < 8$ , both firms commit to a uniform price. In contrast, with asymmetric firms, as in the present paper, only the low quality firm commits to a single price regardless of how refined customer information is. Furthermore, the high quality firm becomes increasingly better off, while

<sup>11</sup> In the rest of the paper, we assume that the cost of acquiring information is sufficiently small and we ignore it.

<sup>12</sup> If information is costless,  $F=0$ , then  $(I,I)$  is also an equilibrium when  $N \geq 4$ . Moreover, if we do not allow firms to play weakly dominated strategies, then  $(I,I)$  becomes the unique equilibrium when  $N \geq 8$ . This is because in this case NI becomes firm L's weakly dominated strategy.

the low quality firm increasingly worse off as the precision of consumer information improves (see Fig. 2).<sup>13</sup>

### 3.3. Firms choose their product qualities (stage 1)

We know from the analysis so far that (I,NI) is the unique equilibrium (provided that the cost of information is not too high) for any value of  $\Delta = s_H - s_L$ . Therefore, the firms' profits can be written as,

$$\pi_H^{I,NI}(N) = (s_H - s_L) \left[ \frac{9N^2 - 3N + 2}{18N^2} \right] - \frac{s_H^2}{2} \quad \text{and} \quad \pi_L^{I,NI}(N) = \frac{(s_H - s_L)}{9N^2} - \frac{s_L^2}{2}.$$

It then follows easily that the equilibrium levels of product qualities are,

$$s_H = \frac{9N^2 - 3N + 2}{18N^2} \quad \text{and} \quad s_L = 0 \quad (5)$$

We have also examined a *global* deviation in product quality, by which the low quality firm increases its quality drastically and becomes the high quality firm. Such a deviation is unprofitable for all  $N$ . On the other hand, the high quality firm clearly has no incentive to become the low quality firm.

Note that  $s_H$  is an increasing function of the consumer information precision and it asymptotically approaches  $1/2$ .<sup>14</sup> Furthermore, it can be readily verified that  $\pi_H^{I,NI}(N)$  monotonically increases and  $\pi_L^{I,NI}(N)$  monotonically decreases with  $N$  (as in Fig. 2) after we substitute the equilibrium product qualities from Eq. (5) into the profit functions.

### 3.4. Welfare

We evaluate the welfare implications of the equilibrium outcome (I,NI). Social surplus will be the difference between the maximum possible benefit to the consumers,  $V + (s_H/2)$  (i.e., when all consumers purchase their favorite brand), and the loss from part of the first

<sup>13</sup> The limit of our game, when  $N \rightarrow \infty$ , is consistent with the result obtained in Shaffer and Zhang (2002). In our model, both the marginal cost of targeting consumers and the horizontal differentiation component are zero (i.e.,  $z=0$  and  $\mathcal{L}_B=0$  in Shaffer and Zhang). Given these parameter values Shaffer and Zhang show (Proposition 2, p. 1148) that only the high quality firm offers targeted promotions. Our Proposition 3 then demonstrates that this is not just a limiting result but rather a quite robust phenomenon.

<sup>14</sup> The results that we have derived so far do not change much qualitatively if we allow the market to be uncovered. More specifically, we set  $V=0$  and we were able to obtain closed form solutions (for fixed  $s_H$  and  $s_L$ ) for all subgames, except (NI,I). We then showed that (I,NI) continues to be an equilibrium. However, we could not solve for  $s_H$  and  $s_L$  explicitly as a function of  $N$ . If we assign specific values for  $N$ , numerical solutions can be obtained. Based on these solutions,  $s_H$  is an increasing function of  $N$  and it asymptotically approaches  $1/2$ , exactly as in the covered market case. Moreover,  $s_L$  is strictly positive, is decreasing as a function of  $N$  and it asymptotically approaches 0 ( $s_L > 0$  for finite  $N$ 's is not surprising, given that the market is uncovered). As we mentioned above, in the uncovered market case, closed form solutions for the (NI,I) subgame cannot be derived. Therefore, we cannot conclude with certainty that (I,NI) is a unique equilibrium, although we conjecture that this should be the case, i.e., when the cost of information,  $F$ , is sufficiently small firm H will have an incentive to deviate to I (given that firm L plays I) and therefore (NI,I) is not an equilibrium. This should be the case since the high quality firm (firm H) has an advantage over all consumers and with some pricing flexibility, it should be able to exploit this advantage.

segment,  $[0, x^*]$ , where the consumers who buy from firm L are located. The cutoff point  $x^*$  is equal to  $p_{H1} - p_{L1} = \Delta/3N$ .

Therefore, social surplus is,

$$SS = V + \int_0^1 \theta_{SH} d\theta - \int_0^{\frac{\Delta}{3N}} \frac{x}{\Delta} dx = V + \frac{s_H}{2} - \frac{s_H}{18N^2}.$$

Consumer surplus is the difference between social surplus and joint profits, i.e.,

$$CS = SS - \pi_H^{I,NI} - \pi_L^{I,NI} = V - \left( \frac{5s_H}{18N^2} - \frac{s_H}{6N} - \frac{s_H^2}{2} \right).$$

Using the equilibrium qualities from Eq. (5), it can be easily verified that both social and consumer surplus are monotonically increasing with  $N$ . This differs from the consumer welfare result derived in Liu and Serfes, where consumer welfare exhibits an inverse U-shape with respect to the information precision  $N$ . This implies that in a model of horizontal differentiation moderate precision of consumer information is best for the consumers, while this is not true in a vertical differentiation setup.

#### 4. Concluding remarks

We propose a price discrimination duopoly model with vertically differentiated products to examine the effect of customer-specific information of varying degrees of precision on the equilibrium of a four-stage game. In stage 1, firms choose their product qualities. In stage 2, firms decide whether to acquire consumer information of a given level of precision. We model consumer information as a partition of the characteristics space. A firm who acquires information is able to segment consumers and charge each consumer group a different price. Information of a higher precision is modeled as a partition refinement. Therefore, our modeling framework bridges the gap between the two most studied models in the literature of two-group and perfect price discrimination. In stages 3 and 4 firms compete in prices.

We show that the high quality firm always benefits from acquiring information at the expense of its low quality rival. If the cost of the customer database is below a threshold level, then only the high quality firm acquires information and the low quality firm commits to a uniform price. This is the equilibrium outcome regardless of the level of information precision. The equilibrium profits of the high quality firm monotonically increase and those of the low quality firm monotonically decrease as the precision of information improves. Finally, social and consumer welfare monotonically increase with the precision of information.

#### Acknowledgements

We would like to thank Simon Anderson (the managing editor), two anonymous referees, George Deltas and Vibhas Madan for very helpful comments and suggestions.

We would also like to thank Greg Shaffer for encouraging us to pursue this topic. The usual caveat applies.

## **Appendix A. Supplementary data**

Supplementary data associated with this article can be found, in the online version, at [doi:10.1016/j.ijindorg.2005.01.005](https://doi.org/10.1016/j.ijindorg.2005.01.005).

## **References**

- Anderson, S., de Palma, A., 1988. Spatial price discrimination with heterogeneous products. *Review of Economic Studies* 55, 573–592.
- Banks, J., Moorthy, S., 1999. A model of price promotions with consumer search. *International Journal of Industrial Organization* 17, 371–398.
- Bester, H., Petrakis, E., 1996. Coupons and oligopolistic price discrimination. *International Journal of Industrial Organization* 14, 227–242.
- Chen, Y., 1997. Paying customers to switch. *Journal of Economics and Management Strategy* 6, 877–897.
- Corts, K., 1998. Third-degree price discrimination in oligopoly. *RAND Journal of Economics* 29, 306–323.
- Fudenberg, D., Tirole, J., 2000. Customer poaching and brand switching. *RAND Journal of Economics* 31, 634–657.
- Lederer, P., Hurter Jr., A.P., 1986. Competition of firms: discriminatory pricing and location. *Econometrica* 54, 623–640.
- Liu, Q., Serfes, K., 2004. Quality of information and oligopolistic price discrimination. *Journal of Economics and Management Strategy* 13, 671–702.
- Motta, M., 1993. Endogenous quality choice: price vs. quantity competition. *Journal of Industrial Economics* 41, 113–131.
- Mussa, M., Rosen, S., 1978. Monopoly and product quality. *Journal of Economic Theory* 18, 310–317.
- Rao, R., 1991. Pricing and promotions in asymmetric duopolies. *Marketing Science* 10, 131–144.
- Shaffer, G., Zhang, Z.J., 1995. Competitive coupon targeting. *Marketing Science* 14, 395–415.
- Shaffer, G., Zhang, Z.J., 2000. Pay to switch or pay to stay: preference-based price discrimination in markets with switching costs. *Journal of Economics and Management Strategy* 9, 397–424.
- Shaffer, G., Zhang, Z.J., 2002. Competitive one-to-one promotions. *Management Science* 48, 1143–1160.
- Shaked, A., Sutton, J., 1982. Relaxing price competition through product differentiation. *Review of Economic Studies* 49, 3–14.
- Stole, L., 2003. Price discrimination and imperfect competition. Survey prepared for the *Handbook of Industrial Organization*.
- Thisse, J.F., Vives, X., 1988. On the strategic choice of spatial price policy. *American Economic Review* 78, 122–137.